

Math 170-003
February 11, 2005

Exam 1 Name _____

This test consists of 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

1. Use the limit laws to find the value of each of the following limits.

(5) (a) $\lim_{x \rightarrow 3} (x^2 + 3x - 5)$

(5) (b) $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 2x + 3}}$

(5) (c) $\lim_{x \rightarrow -2} \frac{x^3 - x + 6}{x + 2}$

(8) 2. What is the definition of the expression “the derivative of $f(x)$ at $x = a$.” (N.B. this definition involves a limit!)

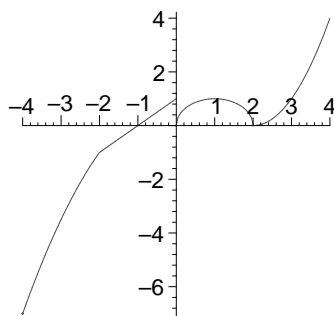
- (7) 3. Suppose we have three functions. $f(x) = x^2 + 3x - 2$ and $g(x) = \sqrt{x^4 + x^2 + 1} - 3$, and an unspecified $h(x)$ about which you know two things:
1. if $-1 < x < 0$, then $f(x) \leq h(x) \leq g(x)$ and
 2. if $0 < x < 1$ then $g(x) < h(x) < f(x)$

What is the value of $\lim_{x \rightarrow 0} h(x)$ and why?

- (10) 4. If $2x + 3y = 7$ is the equation of the tangent to the graph of a function $f(x)$ at $x = 2$, then what are the values of $f(2)$ and $f'(2)$? Why?

- (10) 5. Use the definition of the derivative to find $f'(4)$ if $f(x) = \sqrt{x}$

6. The function f has the graph shown.



- (5) (a) What is the value of $\lim_{x \rightarrow 0^-} f(x)$
- (5) (b) What is the value of $\lim_{x \rightarrow 0^+} f(x)$
- (5) (c) Where does f fail to be continuous?
- (5) (d) Where does f fail to be differentiable?
- (5) 7. Use the intermediate value theorem to show that $1 < \sqrt{3} < 2$.

(10) 8. If $f(x) = \frac{x+3}{x-2}$, find a formula for $f^{-1}(x)$.

(5) 9. Solve the following for x

$$y = e^{2x+3} - 7$$

(10) 10. Give an ϵ - δ proof that

$$\lim_{x \rightarrow 3} \frac{1}{3}x - 5 = -4$$