Math 170-004
September 23, 2005

Exam 1

Name _____

This test consists of 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(8) 1. What is the "ε-δ" definition of a limit? This starts with

$\lim_{x \to a} f(x) = L$ means

2. Use the limit laws to find the value of each of the following limits.

(8) (a) $\lim_{x \to -1} \dfrac{|x+2|}{x+2}$

(8) (b) $\lim_{x \to 3} \dfrac{\sqrt{x+22} - 5}{x-3}$

(8) (c) $\lim_{x \to 4} \dfrac{\frac{1}{x} - \frac{1}{4}}{x-4}$

(8) 3. What value of δ should you use if you want $\mid \frac{1}{x} - \frac{1}{2} \mid < .01$ whenever $0 < \mid x - 2 \mid < \delta$?

(10) 4. You are given $f(2) = 7$ and

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 6$$

What is an equation of the tangent to the graph of f at the point when $x = 2$?

(10) 5. Use the definition of the derivative to find $f'(2)$ if $f(x) = x^2 + 6x - 5$

(5) 6. For what value of c is the function f continuous where f is defined as

$$f(x) = \begin{cases} x^2 + cx + 2 & \text{if } x \leq 3 \\ 4 - cx & \text{if } x > 3 \end{cases}$$

(10) 7. Prove

$$\lim_{x \to 0^+} \sqrt{x} e^{\sin(\pi/x)} = 0$$

(10) 8. (No calculator answers!) If $\sin(x) = 3/4$ and $-\pi/2 \leq x \leq \pi/2$, what is the value of $\cos^2(x) - \sin^2(x)$

(5) 9. Solve the following for x

$$\ln(x) + \ln(x-1) = 1$$

(10) 10. Give an ϵ–δ proof that

$$\lim_{x \to 2}(-2x + 6) = 2$$