

This test consists of 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

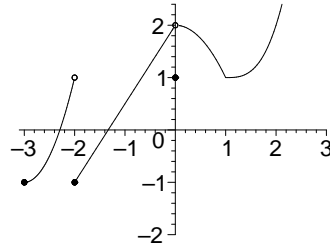
1. Use the limit laws to algebraically compute the following limits: (Numerical arguments are not acceptable.)

(10) (a)  $\lim_{x \rightarrow 5} \frac{\sqrt{x^2 + 144}}{x + 5}$

(10) (b)  $\lim_{x \rightarrow \infty} \frac{3x + 4}{\sqrt{x^2 + 3x + 4} + x}$

(10) (c)  $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x - 3}$

2. The complete graph of a function  $g$  is shown.



Use this graph to answer the following:

(4) (a)  $\lim_{x \rightarrow -3^+} g(x)$

(4) (b)  $\lim_{x \rightarrow 0} g(x)$

(4) (c)  $\lim_{x \rightarrow -2} g(x)$

(4) (d) Where is  $g$  discontinuous?

(4) (e) Where does  $g$  fail to be differentiable?

(10) 3. Find an inverse function for  $f(x) = e^{2x} + 1$

(10) 4. What is the exact value of  $\sin(\cos^{-1}(\frac{2}{3}))$

(10) 5. Suppose  $f(2) = 5$  and  $f'(2) = -2$ . What is an equation of the tangent line to the graph of  $f$  when  $x = 2$ .

(10) 6. One of the consequences of the Intermediate Value Theorem is

*If  $f$  is continuous on an interval  $[a, b]$  and if  $f$  is never zero in  $[a, b]$ , then  $f$  is always positive or always negative on  $[a, b]$ .*

Explain why this is a consequence of the Intermediate Value theorem. (This requires an essay answer.)

7. Use the definition of the derivative to find  $f'(4)$  if  $f(x) = \sqrt{x+5}$

(10) 8. Give an  $\epsilon$ - $\delta$  proof of the statement

$$\lim_{x \rightarrow 5} (-4x + 4) = -16$$