

MATH 170
February 16, 2001

Exam 1 Name _____

This test consists of 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. When you finish the exam, you may hand in your paper and quietly leave.

(10) 1. State the ϵ - δ definition of “the limit of $f(x)$ as $x \rightarrow a$ is L ”

(10) 2. Use the ϵ - δ definition to prove

$$\lim_{x \rightarrow 2} (3x + 4) = 10$$

3. Evaluate the following limits:

(8) (a) $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9}$

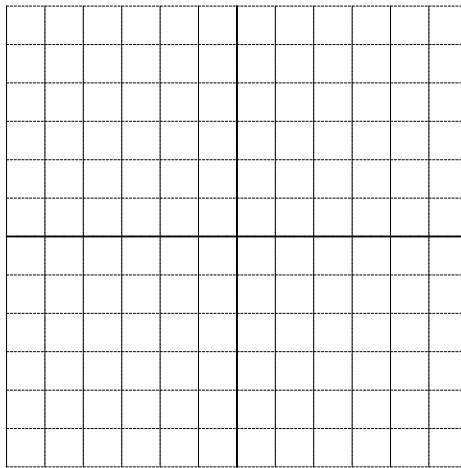
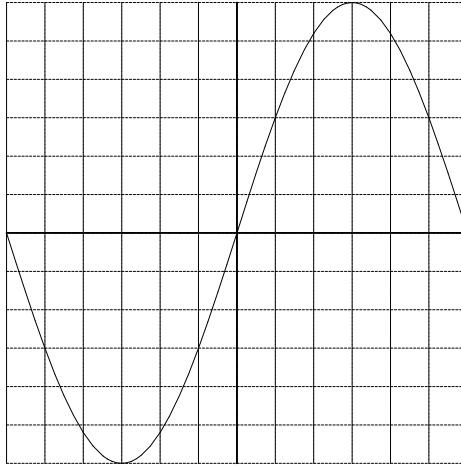
(8) (b) $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x}$

(8) (c) $\lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 2x})$ (Hint: treat as $\frac{x + \sqrt{x^2 + 2x}}{1}$ and rationalize the numerator first.)

(8) (d) $\lim_{x \rightarrow \infty} \frac{7x^3 + 4x}{2x^3 - x^2 + 3}$

- (10) 4. Use the intermediate value theorem to show that $2x^3 + x^2 + 2 = 0$ has a solution in the interval $(-2, -1)$
- (8) 5. What is the definition of the “derivative of f at $x = a$ ”. (N.B. This definition involves a limit.)
- (10) 6. Use the definition of the derivative to find an equation of the tangent to the graph of $f(x) = x^2 + 3x - 2$ at the point $(2, 8)$.

- (10) 7. The graph of a function f is shown below. Sketch a graph of f' on the grid below the graph of f .



- (10) 8. Find a formula for the inverse function of

$$f(x) = \frac{3x - 5}{2x + 4}$$

9. (10 point Extra Credit) Give an ϵ - δ proof of

$$\lim_{x \rightarrow 2} x^2 = 4$$