

This test consists of 4 pages, all different and none intentionally left blank.

Take a minute *right now* to ensure that you have all 4 of these pages. In order to receive credit for your answers, you must show your work!!

1. (10 points) State the ϵ - δ definition of $\lim_{x \rightarrow a} f(x) = L$.

2. (10 points) Use the ϵ - δ definition to prove

$$\lim_{x \rightarrow 2} -2x + 3 = -1$$

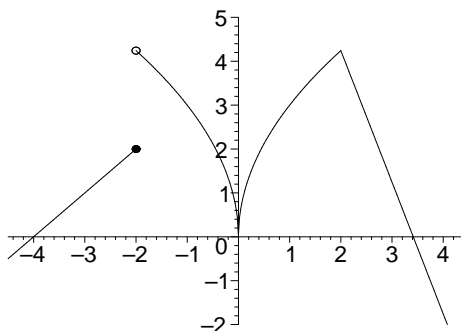
3. (5 points each) If $\lim_{x \rightarrow a} f(x) = 5$ and $\lim_{x \rightarrow a} g(x) = -2$, find each of the following limits:

(a) $\lim_{x \rightarrow a} (f(x)g(x))$

(b) $\lim_{x \rightarrow a} (3f(x) - 5g(x))$

4. (10 points) What is the maple command to draw the graph of $f(x) = x^2$ over the interval $[-2, 2]$?

5. The graph of f is shown below.



Use the graph to find the following limits: (3 points each)

(a) $\lim_{x \rightarrow 0} f(x)$

(b) $\lim_{x \rightarrow 2^-} f(x)$

(c) $\lim_{x \rightarrow 2^+} f(x)$

(d) $\lim_{x \rightarrow -2^-} f(x)$

(e) $\lim_{x \rightarrow -2^+} f(x)$

(f) $\lim_{x \rightarrow 2} f(x)$

6. (5 points each) For each of the following, find the indicated limit, or show it does not exist. Be sure to show your computations.

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 + 2x - 8}$

(b) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

(c) $\lim_{x \rightarrow -1} \frac{|x+1|}{x+1}$

(d) $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$

7. (10 points) Use the squeeze theorem to prove $\lim_{x \rightarrow 0} x^2 \cos(2x) = 0$
8. (10 points) Use the intermediate value theorem to show that $2x^3 - 4x^2 + 5x - 4 = 0$ has a solution in the interval $[1, 2]$
9. (10 points extra credit.) Give examples of two functions, f and g so that neither $\lim_{x \rightarrow 0} f(x)$ nor $\lim_{x \rightarrow 0} g(x)$ exist, but $\lim_{x \rightarrow 0} (f(x)g(x))$ does exist.