M170–003
February 12, 1999 TEST #1 Name _____

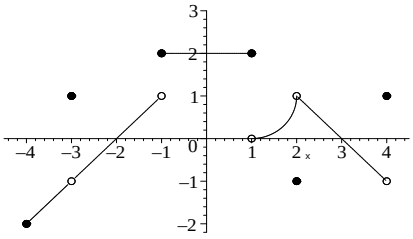
This test consists of **4** pages, all different and none intentionally left blank.
Take a minute *right now* to ensure that you have all 4 of these pages.
In order to receive credit for your answers, you must show your work!!

1. (10 points) State the $\varepsilon - \delta$ definition of a limit.

2. (10 points) Using the $\varepsilon - \delta$ definition, prove the $\lim_{x \to 2}(2x + 3) = 7$

3. State the definition of "f is continuous at $x = a$".

4. The following is the graph of a function f. Using that graph, answer the questions below the graph.



a. $\lim_{x \to 0} f(x)$

b. $\lim_{x \to -3} f(x)$

c. $\lim_{x \to -1} f(x)$

d. $\lim_{x \to 2^-} f(x)$

e. $\lim_{x \to 2^+} f(x)$

f. $\lim_{x \to 2} f(x)$

g. $\lim_{x \to -2^-} f(x)$

h. $\lim_{x \to 1^+} f(x)$

5. Evaluate the following limits.

(a) $\lim_{x \to 1} \dfrac{x^4 - 1}{x - 1}$

(b) $\lim_{x \to +\infty} \sqrt[3]{\dfrac{3x^7 - 4x^5}{2x^7 + 1}}$

(c) $\lim_{x \to 3^+} \dfrac{x}{x - 3}$

(d) $\lim_{x \to 4} \dfrac{4 - y}{2 - \sqrt{y}}$

(e) $\lim_{x \to \infty} (\sqrt{x^2 - 3x} - x)$

6. Let

$$f(x) = \begin{cases} x^2 + 2x - 3 & x \leq 0 \\ -3(1-x)^3 & x > 0 \end{cases}$$

Prove that f is continuous at $x = 0$. (This requires limits, but not ϵ's and δ's)

7. Prove that the equation $x = \cos(x)$ has at least one solution in the interval $[0, \frac{\pi}{2}]$

8. Suppose that $f(x)$ is a function with the property that $1 - x^2 \leq f(x) \leq 1 + x^2$. What can you say about $\lim_{x \to 0} f(x)$, and why?