

MATH275-002
Eleventh Written Assignment
Due at Class Time
Monday, 15 November

- (1) Evaluate

$$\int_C \mathbf{F}(x, y) \cdot d\mathbf{r}$$

where $\mathbf{F}(x, y) = (x + y)\mathbf{i} + (y^2 - x)\mathbf{j}$ and C is the path that starts at $(-1, 0)$, goes along the x -axis to $(1, 0)$ and returns to $(-1, 0)$ along the upper part of the unit circle $x^2 + y^2 = 1$

- (2) Determine if $\mathbf{F}(x, y) = 2xy \sinh(x^2y)\mathbf{i} + x^2 \sinh(x^2y)\mathbf{j}$ is a gradient, then calculate the line integral

$$\int_C \mathbf{F}(x, y) \cdot d\mathbf{r}$$

where $\mathbf{r}(t) = a \cos(t)\mathbf{i} + b \sin(t)\mathbf{j}$ for $0 \leq t \leq 2\pi$

- (3) A wire in the shape of the quarter circle

$$C : \mathbf{r}(t) = a(\cos(t)\mathbf{i} + \sin(t)\mathbf{j}), \quad 0 \leq t \leq \frac{\pi}{2}$$

has varying mass density given by $\lambda(x, y) = k(x + y)$ where k is a positive constant. Find the total mass of the wire and give the coordinates of the center of mass.

- (4) What is the value of

$$\int_C y^2 dx + (xy - x^2) dy$$

if C is the path from $(0, 0)$ to $(2, 4)$ along the path $y^2 = 8x$?

- (5) Let C be the curve traced by $\mathbf{r}(u) = u\mathbf{i} + u^2\mathbf{j} + u^2\mathbf{k}$ for $0 \leq u \leq 1$. Find the value of

$$\int_C yz dx + x^2 dy + xz dz$$