

Math 566  
Spring 2009

7.1

2c)

$$\|\underline{x}\|_2^2 = x_1^2 + x_2^2 + \dots + x_n^2$$

$$\|\underline{x}\|_1^2 = (|x_1| + |x_2| + \dots + |x_n|)^2$$

$$= x_1^2 + x_2^2 + \dots + x_n^2 + 2|x_1x_2| + 2|x_1x_3| + 2|x_1x_4| + \dots + 2|x_{n-1}x_n|$$

$$= x_1^2 + x_2^2 + \dots + x_n^2 + \sum_{j=1}^n \sum_{\substack{l=1 \\ l \neq j}}^n |x_l x_j|$$

$$\geq \|\underline{x}\|_2^2$$

$$\Rightarrow \|\underline{x}\|_1 \geq \|\underline{x}\|_2$$

3b)

$$\lim_{k \rightarrow \infty} k \sin(1/k) = \lim_{k \rightarrow \infty} \frac{\sin(1/k)}{1/k} \quad \begin{matrix} (0) \\ (0) \end{matrix}$$

$$\stackrel{\text{L'H}}{=} \lim_{k \rightarrow \infty} \frac{-1/k^2 \cos(1/k)}{-1/k^2}$$

$$= \lim_{k \rightarrow \infty} \cos(1/k)$$

$$= 1$$

9c.  $\|A\|_F \leq \sqrt{n} \|A\|_2$

Fix  $j$  and define  $x_k = \begin{cases} 0 & \text{if } k \neq j \\ 1 & \text{if } k = j \end{cases}$

$$\text{Then } Ax = \begin{bmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \dots & a_{nj} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{nj} \end{bmatrix}$$

$$\|A\|_2^2 = \max_{\|x\|=1} \|Ax\|_2^2 \geq \sum_{i=1}^n |a_{ij}|^2$$

$$\|A\|_F^2 = \sum_{j=1}^n \sum_{i=1}^n |a_{ij}|^2 \leq \sum_{j=1}^n \|A\|_2^2 = n \|A\|_2^2$$

$$\Rightarrow \|A\|_F \leq \sqrt{n} \|A\|_2$$

7.2  
#14

$$A^T A = A^2 \quad Ax = \lambda x \Rightarrow A^2 x = \lambda^2 x$$

$$\rho(A^T A) = \rho(A)^2 = [\rho(A)]^2$$

$$\|A\|_2 = \sqrt{[\rho(A)]^2} = \rho(A)$$

7.3  
#5 + #7

Please put your results in the form of a table, and show results for each iteration.