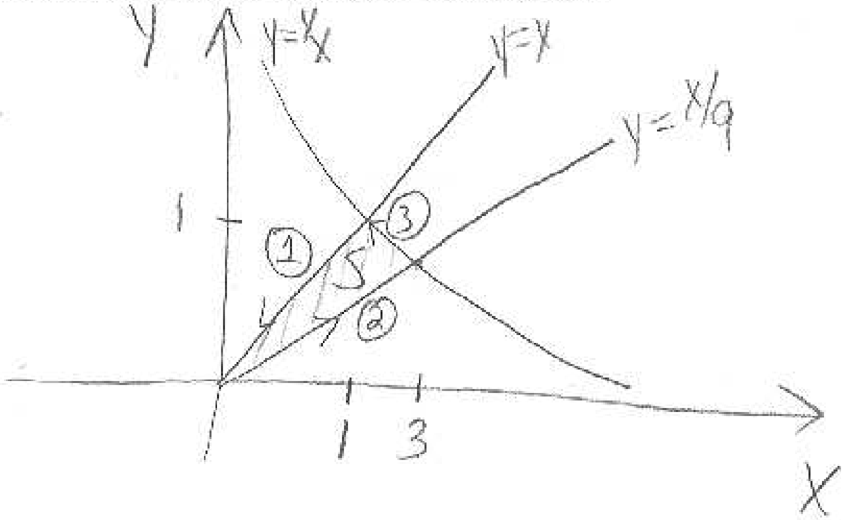
MATH 414/514

October 5, 2009

Names Key

1. Sketch the region S in the first quadrant enclosed by $y = x$, $y = 1/x$, and $y = x/9$. Label the points of intersection of the curves along the boundary of S.



2. Parametrize the boundary in a counter clockwise direction.

1 $x=t$ $\quad t=1,\dots,0$
$y=t$

2 $x=t$ $\quad t=0\dots 3$
$y=t/9$

3 $x=t$ $\quad t=3,\dots,1$
$y=1/t$

3. Find three different P and Q such that

$$\int\int_S dA = \int\int_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$

Hint: There are many choices for P and Q – make it simple.

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$$

1. $\frac{\partial Q}{\partial x} = 1 \quad \frac{\partial P}{\partial y} = 0 \quad \underline{F} = [0, x]^T$
2. $\frac{\partial Q}{\partial x} = 0 \quad \frac{\partial P}{\partial y} = -1 \quad \underline{F} = [-y, 0]^T$
3. $\frac{\partial Q}{\partial x} = \frac{1}{2} \quad \frac{\partial P}{\partial y} = -\frac{1}{2} \quad \underline{F} = [-\frac{1}{2}y, \frac{1}{2}x]^T$

4. Apply one of your choices for P and Q in 3, and your parametrization of ∂S, to Green's theorem so that you can find the area of S using a line integral.

$$\underline{F} = [0, x]^T$$

$$\iint_S dA = \int_{\partial S} \underline{F} \cdot d\underline{x} = \int_{\partial S} \underline{F}(\underline{g}(t)) \cdot \underline{g}'(t) dt$$

$$= \int_1^0 [0, t] \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} dt + \int_0^3 [0, t] \cdot \begin{bmatrix} 1 \\ 1/9 \end{bmatrix} dt$$

$$+ \int_3^1 [0, t] \cdot \begin{bmatrix} 1 \\ -1/t^2 \end{bmatrix} dt$$

$$= \int_1^0 t\,dt + \int_0^3 \tfrac{1}{9} t\,dt + \int_3^1 -\tfrac{1}{t}\,dt$$

$$= \tfrac{1}{2}t^2\Big]_1^0 + \tfrac{1}{18}t^2\Big]_0^3 - \ln|t|\,\Big]_3^1$$

$$= -\tfrac{1}{2} + \tfrac{9}{18} + \ln|3| = \ln 3$$