Math 414/514
Fall 2009

4.1
#7

Since $f(x_0) > 0$ and f is continuous at x_0 there is an interval $[c,d] \subset [a,b]$ containing x_0 so that

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^d f(x)dx + \int_d^b f(x)dx$$

$$\geq \int_c^d f(x)dx \qquad \text{by Theorem 4.9C}$$

$$\geq \int_c^d f(x_0)dx = f(x_0)(d-c) > 0$$

#8

a) $P = \{x_0, \dots, x_J\}$ $\quad P' = \{x_0/c, \dots, x_J/c\}$

$$\sup_{[x_{j-1},x_j]} f(x) = \sup_{[x_{j-1}/c,\, x_j/c]} f(cx) \qquad \inf_{[x_{j-1},x_j]} f(x) = \inf_{[\frac{x_{j-1}}{c},\, \frac{x_j}{c}]} f(cx)$$

$$s_P f = \sum_1^J m_j (x_j - x_{j-1}) = c\sum_1^J m_j \left(x_j/c - \tfrac{x_{j-1}}{c}\right) = c\, s_{P'} f$$

Similarly $S_P f = c S_{P'} f \Rightarrow \int_a^b f(x)dx = c\int_{a/c}^{b/c} f(cx)dx$

b) and c) are similar with different P'

4.2

#5

Rectangles are either 1. contained in S^{int}

2. intersect ∂S

3. do not intersect $\bar{S}$

The sum of those in 1. is $\underline{A}(S^{int}) = \bar{A}(S^{int})$

" " 2. is $\bar{A}(\partial S)$

$1+2 \Rightarrow \bar{A}(\bar{S}) = \underline{A}(S^{int}) + \bar{A}(\partial S)$

Exercise 3 $\Rightarrow \bar{A}(\bar{S}) = \underline{A}(S) + \bar{A}(\partial S)$

4 $\Rightarrow \bar{A}(S) = \underline{A}(S) + \bar{A}(\partial S)$