MATH 333

Forced Equations and Resonance
Lab 5
March 12, 2008

Consider the model of the mass-spring system with periodic forcing:

$$mx'' + cx' + kx = F_0 \cos \omega t \tag{1}$$

where m is the mass of the body attached to the spring, c is a damping coefficient, k is the spring constant, $\omega_0 = \sqrt{k/m}$ is the natural frequency of the system, F_0 is amplitude of the forcing, and ω is the frequency of the external forcing. We can find the exact solution of this differential equation analytically, however, in this lab we will find the numerical solution and study the effect of the forcing term on the behavior of the solution. Use the following script and function to answer the questions in the Exercises.

```
m = ?; c = ?; k = ?; F0 = ?; omega = ?;
param = [m,c,k,F0,omega];
x0 = ?; v0 =?;
[t,X]=ode45(@freson,[0,6*pi],[x0,v0],[],param);
x = X(:,1), v=X(:,2);
figure(1);clf ;hold on; plot(t,x,'b-');
```

```
function dXdt=freson(t,X,param)
x = X(1); v = X(2);
m = param(1); c = param(2); k = param(3); F0=param(4); omega = param(5);
dXdt=[v ; ?? ]
```

<u>Exercises</u>
Due Monday March 17

1. Use the above script and function to solve (1) with $m = 1$, $c = 0$, $k = 9$, $F_0 = 80$ and $\omega = 5$, $x(0) = 0$, and $x'(0) = 0$. You will need to fill in the ??, and you should get the same picture as Figure 3.6.2 on page 214 of your text.
   (a) Hand in a plot of your solution with labels and a title.
   (b) What are the exact amplitude, frequency and period of the external force?
   (c) What are the exact natural frequency and period of the system?
   (d) Describe the period of the solution x and how it relates to the natural and forcing frequencies.
2. Use the above script and function to solve (1) with $m = 0.1$, $c = 0$, $\omega_0 = 55$, $F_0 = 50$ and $\omega = 45$, $x(0) = 0$, and $x'(0) = 0$. You should get the same picture as Figure 3.6.3 on page 215 of your text.
   (a) Hand in a plot of your solution with labels and a title.

(b) What is the exact period of the fast oscillation? What is the exact period of the slow oscillation?

(c) How do the periods of the fast and slow oscillations change as ω moves closer to ω_0? How do they change as ω moves away from ω_0?

3. Use the above script and function to solve (1) with $m = 0.1$, $c = 0$, $\omega_0 = 55$, $F_0 = 50$ and $\omega = 55$, $x(0) = 0$, and $x'(0) = 0$.

   (a) Hand in a plot of your solution with labels and a title.

   (b) Explain what happens. What is the maximum amplitude?

4. Use the above script and function to solve (1) with $m = 0.1$, $c = 5$, $\omega_0 = 55$, $F_0 = 50$, $x(0) = 0$, and $x'(0) = 0$. Vary $\omega = 5, 45, 55, 75$. Explain what happens as you vary ω by comparing the amplitude and frequency of the solution x in each case. Which frequency ω yields the maximum amplitude in the solution? How does this frequency compare to ω_0?