

MAT 333

Pendulum Lab 5 March 5, 2008

Consider the model of the motion of a pendulum with a second order differential equation

$$\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + \omega_0^2 \sin \theta = 0, \quad (1)$$

where c is a damping coefficient, $\omega_0^2 = g/L$, g is gravitational acceleration, and L is the length of the pendulum. This is a nonlinear differential equation with no analytical solution. In this lab we will

- Determine the effect of varying the parameters in the numerical solution.
- Determine the effect of “linearizing” the differential equation.

Use the following script and function to answer the questions in the Exercises.

```
g = 9.81; L = ??; omega = sqrt(g/L); c = ??;  
theta0 = ??; v0 = ??;  
[t,X]=ode45(@fpend,[0,20],[theta0,v0],[],omega,c);  
theta = X(:,1); v=X(:,2);  
figure(1);plot(t,theta*180/pi,'b-');  
grid on
```

```
function dXdt=fpend(t,X,omega,c)  
theta = X(1); v = X(2);  
dXdt=[v ; ??? ];
```

Exercises

Due Monday March 10

1. Use the above script and function to solve (1) with $L = 0.5$ m, $c = 0$, $\theta(0) = \pi/4$ rad, and $\theta'(0) = 1$ rad/s. You will need to fill in the ??.
 - (a) Hand in a plot of the numerical solution of the initial value problem with labels and a title.
 - (b) What is the period of the motion?
 - (c) Will the pendulum come to rest? Why?
 - (d) What is the maximal angle the pendulum makes with the vertical?
 - (e) Does the mass m affect the plot?
2. Try the following variations on the initial conditions. You do not need to hand in a plot for each case however, comment on the behavior of the pendulum in each case. Are any of the behaviors realistic? Are any of them not realistic? If so, what is the explanation of the unrealistic behavior?

- (a) $\theta(0) = 0^\circ$, $\theta'(0) = 2$ rad/s.
 (b) $\theta(0) = 10^\circ$, $\theta'(0) = 8$ rad/s.
 (c) $\theta(0) = \frac{\pi}{2}$ rad, $\theta'(0) = 0$ rad/s.
 (d) $\theta(0) = 180^\circ$, $\theta'(0) = 30$ rad/s.
 (e) $\theta(0) = 360^\circ$, $\theta'(0) = 0$ rad/s.
 (f) $\theta(0) = -7\pi$ rad, $\theta'(0) = 0.03$ rad/s.
 (g) $\theta(0) = -7\pi$ rad, $\theta'(0) = 0.01$ rad/s.
3. If $m = 1$, $L = 0.5$, $c = 0$, $\theta(0) = \frac{\pi}{2}$, and $\theta'(0) = 0$, plot the quantity $E = \frac{1}{2}mL \left(\frac{d\theta}{dt}\right)^2 + mg(1 - \cos\theta)$ as a function of time. What do you observe?
4. Show analytically that $\frac{dE}{dt} = 0$.
5. Plot v vs. θ . How does the curve behave?
6. Now consider a small damping coefficient c in (1). Experiment with the following cases below, using $c = 1$ and $L = 0.5$ m. Comment on the behavior of the pendulum in each case.
- (a) $\theta(0) = 45^\circ$, $\theta'(0) = 1$ rad/s.
 (b) $\theta(0) = 180^\circ$, $\theta'(0) = 0$ rad/s.
 (c) $\theta(0) = 180^\circ$, $\theta'(0) = 10$ rad/s.
7. Create a new function `flpend(t,Y,omega,c)` for the linear case
- $$\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + \omega_0^2\theta = 0. \quad (2)$$
8. Consider the undamped case with $c = 0$ and $L = 0.5$ m, $\theta(0) = \frac{\pi}{2}$, and $\theta'(0) = 0$. Hand in one plot with both linear (2), and non-linear (1) solutions on the same graph.
9. Experiment with the same initial conditions as in 2. and summarize what happens.