MAT 333

Mass Spring System
Lab 5
February 27, 2008

Exercises
Due Monday March 3

Download the GUI masspring.m from the course homepage `http://math.boisestate.edu/~mead/m333/s08`. This GUI will help you visualize solutions of the linear second order differential equation

$$mx'' + cx' + kx = 0$$

as the parameters m, c, and k change. Note that m is the mass, c is the damping coefficient and k is the spring constant. Press *solve* to solve the differential equation, and `>` to watch how the spring follows the solution of the differential equation.

1. Input the mass $m = 1$, no damping (i.e. $c = 0$), spring coefficient $k = 1$, and keep the differential equation homogeneous (right hand side = 0). In addition, let $x(0) = 5$, $v(0)$ =0, and solve for $0 \leq t \leq 20$.
   (a) What is the period of the motion?
   (b) When will the mass-spring system come to rest? Why?
   (c) What is the amplitude of the oscillations for x?
   (d) How does the size of the mass m and the stiffness k of the spring affect the motion? (You may want to vary m and k to answer this question.)
2. Now include damping.
   (a) If $c = 0.4$ for what minimal time t_1 will the mass spring system satisfy $|x(t)| < 1$ for all $t > t_1$?
   (b) How does the size of c affect the motion?
   (c) Determine the smallest value of c such that no oscillation appears in the solution.
3. Create the following functions

```
function [t,x,v]=undamped
x0=0.1; v0=0; m=1; k=4 ; omega=sqrt(k/m);
[t,X]=ode45(@fund,[0,10],[x0,v0],[],omega);
x=X(:,1), v=X(:,2);

function dXdt=fund(t,X,omega)
x=X(1); v=X(2);
dXdt=[v ; -omega^ 2*x];
```

   (a) Use the Undamped functions to find x and v as functions of time t. Plot the quantity $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ as a function of time t. What do you observe?

(b) Show analytically that $\frac{dE}{dt} = 0$.

(c) Plot v vs x. Does the curve ever get close to the origin?

4. Create the following functions

```
function [t,x,v]=damped
x0=0.1; v0=0; m=1; k=4 ; c=1; omega=sqrt(k/m); p=c/(2*m);
[t,X]=ode45(@fd,[0,10],[x0,v0],[],omega,p);
x=X(:,1), v=X(:,2);

function dXdt=fd(t,X,omega,p)
x=X(1); v=X(2);
dXdt=[v ;      ???      ];
```

(a) Fill in dv/dt in the damped function dXdt.

(b) Use the damped functions to find x and v as functions of time t. Plot the quantity $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ as a function of time t. What do you observe?

(c) Show analytically that $\frac{dE}{dt} < 0$ for $c > 0$ while $\frac{dE}{dt} > 0$ for $c < 0$.

(d) Plot v vs x. Comment of the behavior of the curve.