MATH 333

Laplace Transform
Lab 9
May 7, 2008

In this lab we will compute the Laplace transform symbolically and the inverse Laplace transform both symbolically and numerically.

Symbolic representation

The command *syms* assigns a variable to be symbolic, *laplace(f)* finds the Laplace transform of a function f, and *pretty(ans)* puts the answer in a more readable form. Try the following commands:

```
>>syms f t;
>>f=t^2;
>>laplace(f)
ans = 2/s^3
>>pretty(ans)
```

The inverse transform can be computed using *ilaplace(F)*. Try the following commands:

```
>>syms F s
>>F=24/(s*(s+8));
>>ilaplace(F)
ans =
6*exp(-4*t)*sinh(4*t)
```

Numerical representation

There are instances where *ilaplace* will not find the inverse transform, for example try using it to find the inverse transform of $F(s) = \dfrac{1}{s(1+e^{-s})(1+e^{-2s})}$.

When *ilaplace* does not give the inverse transform we can still obtain a numerical approximation to it. One Matlab function for this is *invlap.m* which can be found at `http://math.boisestate.edu/~mead/m333/s08/invlap.m`. This function gives a vector of values of f at different times t, rather than a function $f(t)$, which is best viewed with a graph. Use the following script and function with *invlap.m* to numerically evaluate the inverse transform and plot $f(t)$ when *ilaplace* does not work.

```
t=0.1:0.1:120;
f=invlap(@F,t,0);
plot(t,f); grid on; xlabel('t'); ylabel('f(t)');
```

```
function Fv=F(s)
Fv= 1./(s.*(1+exp(-s)).*(1+exp(-2*s)))
```

Exercises

Due Friday May 9

1. Use Matlab to find the Laplace transform of the following functions:
   (a) $f(t) = e^t \cos t$.
   (b) $f(t) = te^{-t}$.
2. Use Matlab to find the inverse transform of the following functions:
   (a) $F(s) = \dfrac{1}{(s+1)/(s+2)}$.
   (b) $F(s) = \dfrac{1}{s^2+1}$.
   (c) $F(s) = \dfrac{1}{s^5}$.
   (d) $F(s) = \ln\left(1 + \dfrac{1}{s^2}\right)$.
   (e) $F(s) = \dfrac{20(1-e^{-\pi s})}{s(s^2+4s+20)(1+e^{-\pi s})}$.