MATH 333

Euler's Method
Lab 4
February 20, 2008

Before completing the Exercises for the lab, please execute the following:

- In the appropriate directory, download myeuler.m from the class web site:
  `http://math.boisestate.edu/~mead/m333/s08/myeuler.m` .
- Use the function myeuler.m to solve the initial value problem:

$$\frac{dy}{dx} = -2xy^2 \qquad y(0) = 1,$$

  when

$$0 \le x \le 2.$$

  - Create a function file f.m for the right hand side of the differential equation

```
function yp=f(x,y)
yp=-2*x*y^2;
```

  - Create a function file soln.m for the exact solution $y = \dfrac{1}{1+x^2}$ of the differential equation:

```
function yexact=soln(x,y)
yexact=1/(1+x^2);
```

  - In the command window type >>*[X,Y]=myeuler(0,1,2,4);*
- What did you just do?
  - The approximate solution using Euler's Method is Y (check it by typing >>*Y*), and the solution is approximated only at the points X.
  - A table containing the computed solution, the exact solution, and the error is in the file table.txt. Open it in the editor window.
  - A graph of the computed solution and the exact solution is in Figure 1.
- Increase the number of points at which the solutions is approximated by typing >>*[X8,Y8]=myeuler(0,1,2,8);* or >>*[X16,Y16]=myeuler(0,1,2,16);*, etc.
- Change the initial conditions by typing >>*[X1,Y1]=myeuler(1,0.5,2,4);* (what is the difference between (X,Y) and (X1,Y1)?).
- Increase the size of the domain by typing >>*[X2,Y2]=myeuler(0,1,5,4);* (what is the domain?).

Exercises
Due Friday February 22

1. Consider the Initial Value Problem

$$y' = -2y, \qquad y(0) = 3.$$

   (a) Draw the direction field. What is the expected behavior of the solution as t gets larger?

   (b) Determine the exact solution of the IVP.

   (c) Using *myeuler.m*, determine the Euler approximations in the interval $0 \leq t \leq 10$ using $N = 4, 32, 128$ steps. Hand in one plot with $N = 32$, $N = 128$ and the exact solution all in the same figure (with labels). In addition hand in one page with 3 tables, each representing $N = 4, 32, 128$. What do you observe for small values of N (i.e. large h)? Explain the behavior of the approximation obtained for large N using the direction field.