

MATH 333

Systems of Differential Equations Worksheet April 11, 2008

Please work in groups of 2-4 people, and hand in your answers the following questions on separate paper.

1. Let

$$\mathbf{x} = \begin{bmatrix} 3e^{2t} \\ 2e^{2t} \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} e^{-5t} \\ 3e^{-5t} \end{bmatrix}$$

- (a) Verify that \mathbf{x} and \mathbf{y} are linearly independent.
- (b) Verify that \mathbf{x} and \mathbf{y} are solutions of

$$\begin{aligned} x' &= 4x - 3y \\ y' &= 6x - 7y \end{aligned}$$

2. Consider the initial value problem

$$\frac{d\mathbf{x}}{dt} = \mathbf{P}\mathbf{x}$$

with

$$\mathbf{P} = \begin{bmatrix} 3 & -1 \\ 5 & -3 \end{bmatrix},$$

and

$$\mathbf{x}_1(0) = 5 \quad \mathbf{x}_2(0) = -3.$$

- (a) Verify that \mathbf{x}_1 and \mathbf{x}_2 are linearly independent, where

$$\mathbf{x}_1 = e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_2 = e^{-2t} \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

- (b) Verify that these values of \mathbf{x}_1 and \mathbf{x}_2 are solutions of the differential equation.
- (c) Find the solution of the initial value problem.