MAT 333 SECTIONS 001 AND 002 SPRING 2001

Graded Homework for Chapter 7

7.1 (a) Find the Laplace transform of $f(t) = e^{(a+bi)t}$ and $g(t) = e^{(a-bi)t}$.

(b) Show that

$$e^{at}\cos(bt) = \frac{e^{(a+bi)t} + e^{(a-bi)t}}{2}.$$

(c) Use the identity in (b) to show that

$$\mathcal{L}(e^{at}\cos(bt)) = \frac{s-a}{(s-a)^2 + b^2}.$$

7.2 Use the Laplace transform to find the solution of the differential equation

$$y'' - 2y' - 3y = 0,$$

satisfying the initial conditions

$$y(0) = y_0 \qquad y'(0) = y_1.$$

7.3 (a) Use convolutions to find the inverse Laplace transform of $\frac{1}{s^2(s^2-1)}$ and $\frac{1}{s(s^2-1)}$.

(b) Use your results in part (a) to solve

$$y'' - y = g(t),$$

with the initial conditions $y(0) = 2$, $y'(0) = -1$, and

$$g(t) = \begin{cases} 0 & 0 \le t < 1 \\ t & t \ge 1. \end{cases}$$

7.4 Consider

$$x'' + 4x = g(t),$$

with the initial conditions $x(0) = 0$, $x'(0) = 0$, and

$$g(t) = \begin{cases} 0 & 0 \le t < 5 \\ (t-5)/5 & 5 \le t < 10 \\ 1 & t \ge 10. \end{cases}$$

(a) Find the solution.

(b) Graph the forcing function $g(t)$ and the solution $y(t)$ (on separate graphs). Are g and g' continuous? If not, where do the discontinuities occur? How does this effect the solution?

7.5 (a) Solve

$$2y'' + y = 4\delta(t - \pi),$$

where $y(0) = 0$ and $y'(0) = 1$.

(b) Give a physical interpretation of the solution in terms of a mass on a spring.