MAT 333 SECTIONS 001 AND 002 SPRING 2001

Graded Homework for Chapter 2

2.1 Text section 2.1, #27. Reason geometrically, symbolically AND numerically. Explain all three methods clearly: e.g. hand in the slope fields and table of successive values of t.

2.2 The logistic model has been applied to the natural growth of the halibut population in certain areas of the Pacific Ocean. Let y, measured in kilograms, be the total mass, or biomass, of the halibut population at time t. Then

$$\frac{dy}{dt} = ay - by^2.$$

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(a) Scale the solution $y(t)$ by the carrying capacity. Plot it versus time for at least three different initial conditions: $y_0 = \frac{i}{2}(\frac{a}{b})$, $i = 1, \ldots, 3$. What do you observe?

(b) If the initial biomass is $y_0 = .25(\frac{a}{b})$, find a formula for the biomass at any time t.

(c) The parameters in the logistic equation are estimated to have the values $a = 0.71$/year, and $b = 8.82 \times 10^{-9}$/kg-year. Find the biomass 2 years later.

(d) Find the time t for which $y(t) = 0.75(\frac{a}{b})$.

2.3 Consider the differential equation

$$\frac{dx}{dt} = x(x+1)(x-1).$$

(a) What are the equilibrium points?

(b) Determine their stability by drawing the phase line.

(c) Sketch solution curves.

(d) If $x(0) = 2$, what happens to x as t gets very large?

(e) If $x(0) = -2$, what happens to x as t gets very large?

2.4 Consider the differential equation

$$\frac{dx}{dt} = x(2-x) - c.$$

Here c is a constant.

(a) For what values of c does this equation have equilibrium solutions?

(b) Draw the phase lines for $c < 0$, $0 \le c < 1$, $c = 1$, and $c > 1$.

(c) Draw a bifurcation diagram and indicate if, and where a bifurcation occurs.

(d) When $c > 0$, what happens to x as t gets very large if $x(0) = 0$?

2.5 Text section 2.3 #20. Do NOT use formulas (13) and (14) on page 99 of your text. Derive the solution, as is asked in problems 13 and 14.

2.6 Consider the initial value problem

$$\frac{dy}{dt} = e^t \cos y \quad y(0) = 5.$$

(a) What is (are) the equilibrium solution(s)?

(b) *For this question you may use myeuler_1.m, which can be copied from /home/public/mead/ on baron.boisestate.edu, as described in the lab, or can be saved to a file from my web page. It differs from myeuler.m in that it is assumed the exact solution is not known. Note that in Matlab, e^t is typed as* exp(t), *and* $\cos y$ *as* cos(y).

Use Euler's method to solve the initial value problem, and let t be in the interval $[0, 5]$, with $h = 0.1$. Make a table showing the approximate values of y and the corresponding values of t. Then plot the approximate solution found with Euler's method.

(c) Does the solution approach (or try to approach) an equilibrium solution found in (a)? If so, what is it?

(d) What happens if you decrease h to 0.05? What happens if you increase the interval to $[0, 6]$?

(e) Explain possible sources of error. Why does Euler's method have difficulties with this particular problem?