

MATH 275

October 22, 2007

Names _____

In this worksheet you will set up the integral of $\frac{1}{\sqrt{x^2 + y^2 + z^2}}$ over the region E which is bounded by the surfaces $x^2 + y^2 = z^2$ and $x^2 + y^2 = 4y$ in rectangular, cylindrical and spherical coordinates.

1. Rectangular coordinates

(a) Sketch the region bounded by the surfaces $x^2 + y^2 = z^2$ and $x^2 + y^2 = 4y$.

(b) Project the region E onto the $y - z$ plane.

(c) Fill in the limits of integration:

$$\int \int \int \frac{1}{\sqrt{x^2 + y^2 + z^2}} dx dz dy.$$

(d) Project the region E onto the $x - y$ plane.

(e) Fill in the limits of integration:

$$\int \int \int \frac{1}{\sqrt{x^2 + y^2 + z^2}} dz dx dy.$$

2. Cylindrical coordinates

(a) Convert each of the following to cylindrical coordinates:

i. $\frac{1}{\sqrt{x^2 + y^2 + z^2}}$.

ii. $x^2 + y^2 = z^2$.

iii. $x^2 + y^2 = 4y$.

(b) Re-draw the sketch of the region E in 1(a), but label the curves in cylindrical coordinates.

(c) Using cylindrical coordinates set up, but do not evaluate, the integral of $\frac{1}{\sqrt{x^2 + y^2 + z^2}}$ in the region bounded by the surfaces $x^2 + y^2 = z^2$ and $x^2 + y^2 = 4y$.

$$\int \int \int dz dr d\theta.$$

3. Spherical coordinates

(a) Convert each of the following to spherical coordinates:

i. $\frac{1}{\sqrt{x^2 + y^2 + z^2}}$.

ii. $x^2 + y^2 = z^2$.

iii. $x^2 + y^2 = 4y$.

(b) Re-draw the sketch of the region E in 1(a), but label the curves in spherical coordinates.

(c) Using spherical coordinates set up, but do not evaluate, the integral of $\frac{1}{\sqrt{x^2 + y^2 + z^2}}$ in the region bounded by the surfaces $x^2 + y^2 = z^2$ and $x^2 + y^2 = 4y$.

$$\int \int \int d\rho d\phi d\theta.$$