

MATH 275

November 7, 2007

Names _____

1. A closed path C is a path which begins and ends at the same point. What is the value of $\int_C \nabla f \cdot d\mathbf{r}$ over a closed path C ? Prove your answer by using the fundamental theorem of line integrals and assuming that the closed path begins and ends at the point $(\hat{x}, \hat{y}, \hat{z})$.

2. Recall the force vector field that the rubber ducky was swimming in: $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$. You found that the work done on the rubber ducky swimming in a counterclockwise direction for one circular path of radius 2 was 8π . Based on this result, and your answer here for 1., do you think the force field the ducky is swimming in is conservative? Prove your answer by using the definition of a conservative vector field.

3. Find a function f which has gradient vector field $\mathbf{F}(x, y) = \langle y, x + y \rangle$. This function f is called the *potential function* for the vector field.

4. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve from $(2, 1)$ to $(1, 1)$ and \mathbf{F} is the vector in 3.