MATH 275

November 2, 2007

To plot the vector field $\langle -y, x \rangle$ *:*
with(plots);
F:=vector([-y,x]);
fieldplot(F,x=-2..2,y=-2..2, scaling=constrained,axes=frame,thickness=2);

To plot a 2-D parametric curve:
plot($[x(t), y(t), t = t_0..t_1]$,thickness=4);

To plot them together:
p1:=fieldplot(F,x=-2..2,y=-2..2, scaling=constrained,axes=frame,thickness=2):
p2:=plot($[x(t), y(t), t = t_0..t_1]$,thickness=4):
display([p1,p2]);

1. Plot the force vector field $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$.

2. Assume a rubber ducky is moving through the force vector field in 1. with position vector $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$. Plot the path of the rubber ducky for $0 \leq t \leq 2\pi$ together with the force vector field in 1.

3. Will more or less work be done by the force vector field on a rubber ducky that completes one circular path of radius 3? Explain your answer and how it relates to the graph of the force vector field with the circular path.

4. Suppose the rubber ducky follows the same path as in 2., but now in the opposite direction. Give the position vector, and describe how the work done on the rubber ducky changes.

5. Plot the following two force vector fields with the curve given by the position vector $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$ for $-\pi/2 \leq t \leq \pi/2$. From the graphs, estimate if the work done is positive, negative or zero. Explain your answer in words.

   (a) $\mathbf{F}(x, y) =< y, 0 >$.

   (b) $\mathbf{F}(x, y) =< 0, x >$.

6. Plot the force vector field $\mathbf{F}(x, y) =< 1, 0 >$ together with the following three curves defined by vector functions:

$$\begin{aligned} C_1: &\quad \mathbf{r}(t) =< 0.5, t > \quad 1 \leq t \leq 4 \\ C_2: &\quad \mathbf{r}(t) =< t, 1 > \quad 1 \leq t \leq 3 \\ C_3: &\quad \mathbf{r}(t) =< t, 2t > \quad 1 \leq t \leq 2 \end{aligned}$$

   By looking at the graph, arrange the work done along C_1, C_2 and C_3 in ascending order.