MATH 175

Quiz 9

November 10, 2009

Name Key

Please turn in your work on the paper provided.

1. Determine if the following series converge or diverge. Give an explanation for you answer.

(a) (5 pts.) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$.

Ratio Test: $\lim_{n\to\infty} \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \lim_{n\to\infty} \frac{n+1}{10}$

$= \infty$

The series diverges

(b) (5 pts.) $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$.

Ratio Test: $\lim_{n\to\infty} \frac{(n+2)(n+3)}{(n+1)!} \cdot \frac{n!}{(n+1)(n+2)} = \lim_{n\to\infty} \frac{(n+2)(n+3)}{(n+1)(n+1)(n+2)}$

$= \lim_{n\to\infty} \frac{n+3}{n^2+2n+1}$

$= 0$

The series converges

2. (10 pts.) Determine if the following series converges absolutely, converges or diverges

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n+3}$$

Absolute Convergence: $\sum_{n=1}^{\infty}\frac{1}{n+3}=\sum_{n=4}^{\infty}\frac{1}{n}$

this is a divergent p-series $\Rightarrow \sum_{n=1}^{\infty}(-1)^n\frac{1}{n+3}$ does not converge absolutely

Alternating Series test:

① $\frac{1}{n+3}\geq 0$

② $\frac{1}{n+3}\overset{?}{\geq}\frac{1}{n+4}\Rightarrow n+4>n+3$ ✓

③ $\lim_{n\to\infty}\frac{1}{n+3}=0$

The series converges

$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n+3}$