MATH 175
Quiz 10

November 17, 2009

Name Key

Please turn in your work on the paper provided.

Find the series' radius and interval of convergence.

1. (10 pts.) $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2+3}}$.

...o test ...bsolute ...vergence

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{\sqrt{(n+1)^2+3}} \frac{\sqrt{n^2+3}}{x^n} \right| = \lim_{n\to\infty} \left| \frac{\sqrt{n^2+3}}{\sqrt{n^2+2n+4}} x \right|$$

$$= |x| \lim_{n\to\infty} \sqrt{\frac{n^2+3}{n^2+2n+4}}$$

$$= |x| \lim_{n\to\infty} \sqrt{\frac{1+3/n^2}{1+2/n+4/n^2}}$$

$$= |x|$$

Converges if $|x|<1 \Rightarrow \boxed{\text{Radius of convergence} = 1}$

Test the endpoints to determine the interval

$x=1$: $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+3}}$ compare to $\sum \frac{1}{n}$ divergent

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+3}} = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2+3}} = \lim_{n\to\infty} \sqrt{\frac{1}{1+3/n^2}} = 1 > 0$$

both series diverge by limit comparison test.

$x=-1$: $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}}$ does not converge absolutely. Alternating Series test: $\frac{1}{\sqrt{n^2+3}} \overset{?}{\geq} \frac{1}{\sqrt{(n+1)^2+3}} \Rightarrow (n+1)^2+\cancel{3} \geq n^2+\cancel{3}$

and $\lim_{n\to\infty} \frac{1}{\sqrt{n^2+3}} = 0 \Rightarrow$ converges

$\boxed{\text{Interval: } -1 \leq x < 1}$

2. (10 pts.) $\sum_{n=0}^{\infty} \left(\frac{x^2+1}{3}\right)^n$.

Root test: $\lim_{n\to\infty} \frac{x^2+1}{3} = \frac{x^2+1}{3} = \rho$

Converges if $\frac{x^2+1}{3} < 1 \Rightarrow x^2+1<3 \Rightarrow x^2<2$

$\Rightarrow -\sqrt{2} < x < \sqrt{2}$

Radius of convergence $= \sqrt{2}$

$x=\sqrt{2}$: $\sum_{n=0}^{\infty} (1)^n$ diverges by divergence test

$x=-\sqrt{2}$ $\sum_{n=0}^{\infty} (1)^n$ " "

interval of convergence $-\sqrt{2} < x < \sqrt{2}$