MAT 170 Section 005

Review for Final Exam

December 9, 2005

Chapter 4

1. On which intervals is the function $f(x) = x^3 - 3x^2 + 1$ increasing, decreasing, concave up and concave down?

2. Locate the local maximum and minimum of

   (a) $f(x) = x^3 - 3x^2 + 3x - 1$.

   (b) $f(x) = x^4 - 2x^2$.

3. Prove that if f is differentiable and continuous everywhere, and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.

4. Evaluate the following limits.

   (a) $\lim_{x\to\infty} \frac{x^2}{e^x}$

   (b) $\lim_{x\to\infty} x^{\frac{1}{x}}$

   (c) $\lim_{x\to\infty} (\sin x)^{\frac{1}{x}}$

   (d) $\lim_{x\to 0} \frac{1-\cos(2x)}{x^2}$.

   (e) $\lim_{x\to 1} \frac{\tan(\frac{\pi x}{4}) - 1}{\pi(x-1)}$.

   (f) $\lim_{x\to\infty} x^{1/x}$.

5. Investigate the family of functions
$$f(x) = \ln(\sin x + c)$$
   where c is a constant. Do this by answering the following questions and sketching $f(x)$ with 5-10 different values of c (draw all functions on the same graph), so that the effect of different values of c is clear.

   (a) What features do members of this family have in common?

   (b) How do they differ?

   (c) For which values of c is f continuous on $(-\infty, \infty)$?

   (d) For which values of c does f have no graph at all?

   (e) What happens as $c \to \infty$?
   Hint:
   Your analysis will go much smoother if you first determine for all c the (1) periodicity, (2) domain, (3) x and y intercepts, (3) asymptotes, and (4) critical points.

(OVER)

6. Determine the maximum area of a rectangle that can be inscribed in a right-angle triangle with sides 6 and 8 units.

   (i.e. the hypotenuse is $\sqrt{6^2+8^2}$*, one corner of the rectangle is the right-angle of the triangle, and the opposite corner of the former touches the hypotenuse of the latter.)*

7. An open-top box is to have square base and volume 10 ft^3. If the bottom costs 15 dollars per square foot and the sides cost 6 dollars per square foot, find the dimensions that achieve the minimum cost. You must also show that these dimensions actually give rise to the minimum cost.

8. Explain why Newton's method fails when applied to the equation $\sqrt[3]{x} = 0$ with ANY initial approximation $x_1 \neq 0$.