MAT 170 Section 005
Review for Final Exam

December 6, 2005

Please work in groups with no more than four people.

Chapter 2

1. Let $f(x) = \frac{1}{x^2}$. Find the slope of the secant line between those points on the graph for which $x = 1$ and $x = 2$. Then find the slope and equation for the tangent to the graph of f where $x = 2$.

2. During the first 40 seconds of a rocket flight, the rocket is propelled straight up so that in t seconds it reaches a height of $5t^3$ ft.

   (a) What is the average velocity (rate of change) of the rocket during the first 40 seconds?

   (b) What is the average velocity (rate of change) during the first 135 ft. of flight?

   (c) What is the instantaneous velocity (rate of change) of the rocket at the end of 40 seconds?

3. Find an equation of the tangent line to the curve $\sqrt{x} + \sqrt{y} = 3$ at $(4, 1)$.

4. Evaluate the following limits

   (a) $\lim_{x \to 8^-} \dfrac{|x-8|}{x-8}$.

   (b) $\lim_{x \to 8} \dfrac{|x-8|}{x-8}$.

   (c) $\lim_{x \to \infty} e^{-3x}$.

   (d) $\lim_{x \to \infty} \dfrac{\sqrt{1+4x^2}}{4+x}$.

   (e) If $f(x) = x^2 + 5$, find $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

5. Recall the precise definition of limit (for appropriate $f(x)$ and a):

$$\lim_{x \to a} f(x) = L$$

   if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x - a| < \delta.$$

   Consider the case when $f(x) = 6x - x^2$ and $L = 8$.

   (a) Illustrate the definition of limit on a graph using this expression for $f(x)$ and value for L.

   (b) Let $\epsilon = 0.5$. Find a value of δ for which the definition holds.

6. Is the following function continuous? If not, state the points where it is not continuous and explain clearly why it is not continuous there.

$$f(x) = \begin{cases} \sqrt{-x} & \text{if} \quad x < 0 \\ 3 - x & \text{if} \quad 0 \le x < 3 \\ (x-3)^2 & \text{if} \quad x \ge 3 \end{cases}$$

7. Prove that there exists at least one real root of the equation $x^{101} + x^{51} + x - 1 = 0$.