Electrical Resistivity Imaging with Subsurface Boundary Constraints

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Outline

- Electrical Resistivity Tomography (ERT)
- Prior information and joint inversion
- Assessing uncertain prior information
- Results from Boise Hydrogeological Research Site (BHRS)
Boise Hydrogeological Research Site

- Field laboratory on a gravel bar adjacent to the Boise River, 15 km southeast of downtown Boise.
- Consists of coarse cobble and sand. Braided stream fluvial deposits overlie a clay layer at about 20 m depth.

- Difference in retention properties in a lenticular sand feature yields significantly different geophysical properties.
Electrical Resistivity Tomography (ERT)

- 2D grid of observations provides a 3-D inverted model that emphasizes the sand lenticular feature.
- BHRS survey consisted of 12 electrodes spaced 1 meter apart acquired with a dipole-dipole configuration.
- BHRS survey acquired at surface when subsurface achieved saturation.
Ohm’s Law

\[ \nabla \cdot [\sigma(\mathbf{r}) \nabla V(\mathbf{r})] = i \left[ \delta(\mathbf{r} - \mathbf{r}_A) - \delta(\mathbf{r} - \mathbf{r}_B) \right] \]

Model parameters – resistivity \( \rho = \frac{1}{\sigma} \)

Observed data – apparent resistivity \( \rho_a = \frac{2\pi \Delta V}{i \kappa} \)

\( \Delta V \) - electrical potential difference across receiver electrode pair

\( \kappa \) - contains geometrical information about electrodes

\( \mathbf{r}_A, \mathbf{r}_B \) - locations of current source electrodes A and B
Constrained Inversion (Regularization)

\[ \varphi[m] = ||W(F[m] - d)||_2^2 + \alpha^2 ||Lm||_2^2 \]

- \( m \) : model parameters (resistivity)
- \( F[\ ] \) : forward model (Ohm’s Law)
- \( d \) : observed data (apparent resistivity)
- \( \alpha \) : regularization parameter

\[ W = \text{diag}(1/\sigma), \sigma \text{ data error estimates} \]

\[ L = \frac{1}{\Delta x} \begin{bmatrix} -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & -1 & 1 \end{bmatrix} \]
Discontinuous Inversion: Relaxing the Constraint

\[ \varphi[m] = ||W(F[m] - d)||^2_2 + \alpha^2||RLm||^2_2 \]

- \( R = \text{diag}(r_1, \ldots, r_n) \), \( r_i = 0 \) or \( 1 \)
- \( r_i = 0 \rightarrow \) no regularization at discontinuity specified at \( i \)
  - \( \rightarrow \) no smoothness at \( i \)
- Only data informs parameter at discontinuity
Joint Inversion: Incorporating Prior Information

\[ \text{diag}(R) = \]

[Image of a graph showing depth vs. distance with color-coded inverted model]

[Image of a graph showing a pattern with color scale]
Ground Penetrating Radar (GPR)

- GPR survey at BHRS acquired across center of gridded ERT survey.
- GPR sampled line collinear with ERT survey.
- GPR derived boundary gives prior boundary knowledge in the ERT dataset.
Uncertain Boundary Location

Given $n_s$ subregions, which boundary estimate $R$ is best?
Identify Best Boundary Estimate: Data residual

\[ \varphi_d[\hat{m}] = \|W(F[\hat{m}] - d)\|_2^2 \]
\[ \hat{m} = \arg\min_{m} \|W(F[m] - d)\|_2^2 + \alpha^2 \|RLm\|_2^2 \]

As \( \alpha \to \infty \)
- Homogeneous \( \hat{m} \), \( \varphi_d \) not minimized
- Prior R gives \( n_s \) homogeneous subregions
- Variability in \( \varphi_d[\hat{m}(R)] \) due to boundary location

\[ \hat{R} = \arg\min_{R} \{ \lim_{\alpha \to \infty} \varphi_d[\hat{m}(R)] \} \]
Choose $R$ with smallest $\varphi_d$ as $\alpha \to \infty$
Identify Best Boundary Estimate: Subregion variability

- Incorrect R results in smearing near discontinuity
- Identify parameter variability within each inverted subregion
  - Fit $n_s$ Gaussians to histogram of inverted parameters
  - Determine $\sigma$ for each Gaussian
Subregion variability, $n_s=2$

- Improper boundary estimate results in more variability in subregions.
- $R$ that gives inverted parameters with smallest $\sigma$ is best boundary estimate.
ERT Inversion results from BHRS

Proper R

R shifted up

R shifted down
ERT Inversion results from BHRS

Proper R

R shifted left

R shifted right
## Data Residual for BHRS dataset

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\lim_{\alpha \to \infty} \varphi_d[\hat{m}(R)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proper</td>
<td>117,230</td>
</tr>
<tr>
<td>Shifted up</td>
<td>305,592</td>
</tr>
<tr>
<td>Shifted down</td>
<td>251,327</td>
</tr>
<tr>
<td>Shifted left</td>
<td>305,671</td>
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<tr>
<td>Shifted right</td>
<td>305,735</td>
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</table>
Subregion variability for BHRS dataset

<table>
<thead>
<tr>
<th>R</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proper</td>
<td>147</td>
<td>710</td>
<td>100</td>
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<tr>
<td>Shifted up</td>
<td>diverged</td>
<td>diverged</td>
<td>diverged</td>
</tr>
<tr>
<td>Shifted down</td>
<td>431</td>
<td>497</td>
<td>235</td>
</tr>
<tr>
<td>Shifted left</td>
<td>276</td>
<td>3000</td>
<td>449</td>
</tr>
<tr>
<td>Shifted right</td>
<td>216</td>
<td>3000</td>
<td>310</td>
</tr>
</tbody>
</table>
Summary and Conclusions

• Discontinuities (subsurface boundaries) were identified with GPR and input into an ERT inversion algorithm as constraints or prior information.

• Reliability of prior boundary information was evaluated with data residuals and subregion variability.

• In addition to BHRS data set, this approach was tested on 7 different simulated examples. Together with theoretical justifications, we determined that the following are effective:
  – Including boundary information through R in the constraint
  – Identifying best R through data residuals and subregion variability
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Thank You