Final Exam: Math 297

Spring 2008

Start: May 7. Due: May 15, 11:00 a.m.

Instructions.

1. A hardcopy of your answers must be handed in by the due date and time.
2. The document handed in must contain your name on at least the front page.
3. Carefully substantiated answers accompanied by appropriate exposition in grammatically correct English are required. No credit will be given for illegible, unsubstantiated or otherwise unintelligible answers.
4. Although a reference is given for Question 1, it is not necessary to consult this reference to be able to answer this question.
5. This is an examination. As such it is expected to be your own work, done without assistance from other humans.
6. This examination is worth 100 points.

Question 1

In *Uroleptus*, a ciliate, it happens that a macronuclear chromosome contains two genes, call them gene A and gene B. The micronuclear precursors of these two genes have the usual scrambled architecture, and occur on two distinct loci. It is believed that the two loci resulted from a DNA editing event that separated the micronuclear package into two loci. The putative micronuclear version that existed before this event is believed to have the following MDS order:

7 - 9 - 3 - 10 - 12 - 14 - 1 - 2 - 4 - 5 - 6 - 8 - 11 - 13 - 15 - 16

Here, the numbers are the numbers of the MDSs, and the IESs are located at the "-" signs. Gene A consists of the MDSs 1 through 5, while gene B consists of the remaining MDSs. See Figure 1 (B) of [1]

Carefully demonstrate, in single steps that clearly show the products of each step, how this micronuclear precursor can be processed, in the template model, by the three intramolecular operations hi, ld and dlad, to result in a pre-chromosome fragmentation version of the macronuclear chromosome that contains both genes A and B. (20)

Question 2

Consider Turing machines whose tape alphabets consist of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, x. We shall also use these numbers in the following translation table to denote, in tape language, specific programming language symbols used in specifying Turing machine programs:

Program Language Alphabet	Tape language
1	1
2	11
3	111
4	1111
5	11111
6	111111
7	1111111
8	11111111
9	111111111
x	1111111111
$'$	2
$\sqcup$	3
R	4
L	5
q	6
q'	62
q''	622
$\vdots$	$\vdots$
$\rightarrow$	7
(	8
)	9
,	x

The symbols q, q', q'', $\cdots$ are used to denote distinct states of the Turing machine, and the symbol $\sqcup$ denotes, in the programming language, the blank square on the tape.

The program clauses are in the form

(state,symbol)$\rightarrow$(state,symbol,L or R).

The pair (state,symbol) left of the $\rightarrow$ denotes the current state of the machine and the symbol present in the square currently scanned by the reading head. The triplet to the right of the arrow denotes the new state the machine enters, the symbol it writes in the currently scanned square (overwriting any symbol already present) and the L indicates the reading head moves one cell to the left, while the R indicates the reading head moves one cell to the right.

In the interest of saving space these program clauses are represented in tabular form as follows:

State/Symbol	1	2	3	4	5	6	7	8	9	x	⊔
q	(q′,2,R)	·	·	·	·	·	·	·	·	·	·
q′	(q,3,R)	·	·	·	·	·	·	·	·	·	·
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

In this table, the entry in the row labelled "q", and the column labelled "1" is read as

$$(q,1)\rightarrow(q',2,R).$$

In tape language this program clause would read:

86x197862x11x49

Consider the following program of a two-state Turing machine:

	1	2	3	4	5	6	7	8	9	x	⊔
q	(q′,2,R)	(q,3,R)	(q′,1,L)	(q′,5,R)	(q,2,L)	(q′,4,R)	(q,5,L)	(q′,⊔,R)	(q,x,L)	(q′,9,R)	(q,8,L)
q′	(q,3,R)	(q,5,L)	(q,9,R)	(q,2,L)	(q,7,L)	(q,x,L)	(q,x,R)	·	·	·	·

(a) Rewrite this program row-by-row in tape language, starting with the top left entry of the table. (10)

(b) Execute this machine's program on the tape input created in (a), starting in state q, and scanning the leftmost symbol of the input. Determine if the machine halts on this input. If it does halt, display the tape contents, and give the state in which the machine halted, and indicate which cell on the tape is being scanned at this point. (15)

(c) Execute this machine's program on the tape input created in (a), starting in state q′, and scanning the leftmost symbol of the input. Determine if the machine halts on this input. If it does halt, display the tape contents, and give the state in which the machine halted, and indicate which cell on the tape is being scanned at this point. (15)

(d) Rewrite this program column-by-column in tape language, starting with the top left entry of the table. (10)

(e) Execute this machine's program on the tape input created in (d), starting in state q. Determine if the machine halts on this input. If it does halt, display the tape contents, and give the state in which the machine halted, and indicate which cell on the tape is being scanned at this point. (15)

(f) Execute this machine's program on the tape input created in (d), starting in state q'. Determine if the machine halts on this input. If it does halt, display the tape contents, and give the state in which the machine halted, and indicate which cell on the tape is being scanned at this point. (15)

References

[1] S. Kuo, W-J Chang and L.F. Landweber, *Complex germline architecture: Two genes intertwined in two loci*, **Mol. Biol. Evol** 23:1 (2006), 1 - 4.