[redacted]

Sections 5, 6 Very good! 5/5

5.3 Disprove: If a, b, c are positive integers with $a|(bc)$, then $a|b$ or $a|c$.

Counterexample: When $a=6$, $b=3$, $c=4$ then $a|(bc) = 6|(3\cdot4) = 6|12$, but $a|b = 6|3$ and $a|c = 6|4$ are false.

6.8 Prove: $(x \vee y) \to z$ is logically equivalent to $(x \to z) \wedge (y \to z)$

Proof:

x	y	z	$(x \vee y)$	$(x \to z)$	$(y \to z)$	$(x \vee y) \to z$	$\equiv$	$x \to z \wedge y \to z$
T	T	T	T	T	T	T		T
T	T	F	T	F	F	F		F
T	F	T	T	T	T	T		T
T	F	F	T	F	T	F		F
F	T	T	T	T	T	T		T
F	T	F	T	T	F	F		F
F	F	T	F	T	T	T		T
F	F	F	F	T	T	T		T

6.10 Show the following:

a) $x \to y$ is not logically equivalent to $y \to x$.

Proof:

x	y	$x \to y$	$\neq$	$y \to x$
T	T	T		T
T	F	F		T
F	T	T		F
F	F	T		T

6.10 b) $x \to y$ is not logically equivalent to $x \leftrightarrow y$.

Proof:

x	y	$x \to y$	$\not\equiv$ $x \leftrightarrow y$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

c) $x \vee y$ is not logically equivalent to $(x \wedge \neg y) \vee (\neg x \wedge y)$.

Proof:

x	y	$\neg x$	$\neg y$	$(x \wedge \neg y)$	$(\neg x \wedge y)$	$x \vee y$	$\not\equiv$ $(x \wedge \neg y) \vee (\neg x \wedge y)$
T	T	F	F	F	F	T	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	F	F	F

6.14 find an expression that is logically equivalent to $x \leftrightarrow y$ and uses only the basic operations $\wedge, \vee, \neg$ (and prove it).

$x \leftrightarrow y = (x \wedge y) \vee ((\neg x) \wedge (\neg y))$

Proof:

x	y	$\neg x$	$\neg y$	$(x \wedge y)$	$(\neg x \wedge \neg y)$	$x \leftrightarrow y$	$\equiv$ $(x \wedge y) \vee (\neg x \wedge \neg y)$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

[redacted]

8. The following statement is false:
If x, y, z are integers and $x > y$, then $xz > yz$.
Please do the following:

a) find a counterexample.

Counterexample: When $x = 2$, $y = 1$, $z = -1$,
then $x > y = 2 > 1$ but $xz \not> yz = (2 \cdot -1) \not> (1 \cdot -1) = -2 \not> -1$.

b) Modify the hypothesis of the statement by adding a condition concerning z so that the edited statement is true.

If x, y, z are integers and $x > y$ and z is a positive integer, then $xz > yz$.