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Here are more augmented-matrix/GJ/RREF problems. This time be on guard for many-solutions and no-solutions situations.

1 Solve the following system. Announce your final solution in column-matrix form.

$$\begin{aligned} -8x + 3y + 39z &= -37 \\ -3x + y + 14z &= -15 \\ 10x - 3y - 45z &= 53 \end{aligned}$$

2 Use the augmented-matrix/GJ methods to find a column-matrix formula for all solutions of

$$\begin{aligned} -8x - 24y + 3z &= 63 \\ -3x - 9y + z &= 23 \\ 10x + 30y - 3z &= -75 \end{aligned}$$

This system has one free variable, y, and two basic variables, x and z.

3 Consider the points

$$(45, 13) \qquad (-30, 3) \qquad (60, 15).$$

Find an equation of form $Ax + By + C = 0$ for the line (if any) which passes through these three points.

As in the parabola example, use substitution to write a linear system whose unknowns are the parameters A, B, and C. Or maybe C, A, and B.

Use GJ to solve the system and interpret the final result.

4 If you graph the points

$$(2, 3) \qquad (4, -7) \qquad (8, 12)$$

you can see that there is no line through these points. If you blindly follow the method of the previous problem, you will arrive at a linear system which *does* have a solution. How does this system's GJ endgame nevertheless show that there is no line through these three points?

5 Find an equation for the circle through the three points of problem 4.

Hint: $x^2 + y^2 + Ax + By + C = 0$

6 Nilknarf didn't review partial fractions very well. But he's gotten along just fine until he hit

$$\int \frac{16x^2 - 80x + 76}{(x-1)(x+2)(x-3)^2}\, dx.$$

For the integrand here, he has written down the following partial-fractions guess:

$$\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x-3)^2}.$$

Nilknarf goes through the usual "strategic-substitution" method of finding values for the parameters A, B, and C : he puts the guess over a least common denominator and equates numerators:

$$16x^2-80x+76=A(x+2)(x-3)^2+B(x-1)(x-3)^2+C(x-1)(x+2) \quad (1)$$

for which

(i) Strategic substitution $x=1$ yields $A=1$

(ii) Strategic substitution $x=-2$ yields $B=-4$

(iii) Strategic substitution $x=3$ yields $C=-2$

This leads Nilknarf to believe that

$$\frac{16x^2-80x+76}{(x-1)(x+2)(x-3)^2}=\frac{1}{x-1}+\frac{-4}{x+2}+\frac{-2}{(x-3)^2}.$$

which makes for easy integrations.

However, when Nilknarf goes to check his work, he discovers that

$$\frac{1}{x-1}+\frac{-4}{x+2}+\frac{-2}{(x-3)^2}=\frac{-3x^3+\cdots}{(x-1)(x+2)(x-3)^2},$$

not a welcome bit of news. This results in problems for *you*:

(a) Go back to equation (1), multiply out the right-hand side, collect the coefficients of powers of x, equate coefficients of powers of x, and arrive at a linear system of equations in unknowns A, B, and C.

(b) Write down the augmented matrix for the system you obtained just now. Turn the GJ crank as far as it will go, then announce your results.

(c) Set up the *correct* partial-fractions guess for Nilknarf's problem. Use the equate-coefficients method to write down the linear system for the parameters in your partial-fractions guess.

(d) Use GJ to solve the system of equations you derived in the previous problem. Your solution is a sequence of *five* augmented matrices, followed by an announcement of the partial-fractions decomposition.

(e) Evaluate the integral that started this whole mess.

