These are alleged answers. For each error herein, you get extra-credit points for being the first to report it by e-mail.

1 $y'' - 4y' + 4y = \mathbf{Step}(t, 3)$ with $y(0) = 1$ and $y'(0) = -1$.

The initial-value problem transforms to

$$Y(s) = \frac{s-5}{(s-2)^2} + e^{-3s}\frac{1}{s(s-2)^2}$$

so that

$$Y(s) = \left[\frac{1}{s-2} - \frac{3}{(s-2)^2}\right] + \frac{e^{-3s}}{4}\left[\frac{1}{s} - \frac{1}{s-2} + \frac{2}{(s-2)^2}\right]$$

or

$$Y(s) = \mathcal{L}\left(e^{2t} - 3te^{2t}\right) + \frac{e^{-3s}}{4}\mathcal{L}\left(1 - e^{2t} + 2te^{2t}\right)$$

so that

$$y(t) = e^{2t} - 3te^{2t} + \frac{1}{4}\mathbf{Step}(t,3)\left(1 - e^{2(t-3)} + 2(t-3)e^{2(t-3)}\right).$$

For a "**Step**-free" form

$$y(t) = \begin{cases} e^{2t}(1-3t) & \text{if } 0 \leqslant t \leqslant 3 \\ e^{2t}(1-3t) + \frac{1}{4}\left(1 + (2t-7)e^{2(t-3)}\right) & \text{if } 3 \leqslant t \end{cases}$$

2 $\boldsymbol{y'' + 2y' - 3y = \mathbf{Step}(t,4)e^{2t}}$ **with** $\boldsymbol{y(0) = 1}$ **and** $\boldsymbol{y'(0) = 0.}$

Note first the Laplace transform of the right-hand side:

$$\begin{aligned} \mathcal{L}\left(\mathrm{Step}(t,4)e^{2t}\right) &= e^{-4s}\mathcal{L}\left(e^{2(t+4)}\right) \\ &= e^{-4s}e^{8}\mathcal{L}\left(e^{2t}\right) \\ &= e^{8}e^{-4s}\frac{1}{s-2} \end{aligned}$$

Thus the initial-value problem transforms to

$$Y(s) = \frac{s+2}{(s+3)(s-1)} + e^{8}e^{-4s}\frac{1}{(s-2)(s+3)(s-1)}$$

so that

$$Y(s) = \left[\frac{1/4}{s+3} + \frac{3/4}{s-1}\right] + e^{8}e^{-4s}\left[\frac{-1/4}{s-1} - \frac{1/5}{s-2} + \frac{1/20}{s+3}\right]$$

or

$$Y(s) = \mathcal{L}\left(\frac{1}{4}e^{-3t} + \frac{3}{4}e^{t}\right) + e^{8}e^{-4s}\mathcal{L}\left(-\frac{1}{4}e^{t} + \frac{1}{5}e^{2t} + \frac{1}{20}e^{-3t}\right)$$

so that

$$y(t) = \frac{1}{4}e^{-3t} + \frac{3}{4}e^{t} + e^{8}\mathrm{Step}(t,4)\left(-\frac{1}{4}e^{t-4} + \frac{1}{5}e^{2(t-4)} + \frac{1}{20}e^{-3(t-4)}\right).$$

For a "**Step**-free" form

$$y(t) = \begin{cases} \frac{1}{4}e^{-3t} + \frac{3}{4}e^{t} & \text{if } 0 \leqslant t \leqslant 4 \\ \frac{1}{4}e^{-3t} + \frac{3}{4}e^{t} + e^{8}\left(-\frac{1}{4}e^{t-4} + \frac{1}{5}e^{2(t-4)} + \frac{1}{20}e^{-3(t-4)}\right) & \text{if } 4 \leqslant t \end{cases}$$