Sun Dec 12 14:20:35 MST 2004 /m333.fa04/handouts333/t4_333_C13/t4_333_C13

This test has pages 1 – 8. Take a moment to make sure you have them all.

No Calculators Allowed; No Reference Materials; Just You and Your Pencil and Eraser.

1 Fill in the blanks in the following table of functions (in the left column) and their Laplace Transforms (in the right column). $\boldsymbol{H}$ denotes the Heaviside function.

	$\boldsymbol{f(t)}$	$\boldsymbol{F(s)}$
a	$\boldsymbol{t^4}$	
b		$\boldsymbol{\frac{1}{s^4}}$
c	$\boldsymbol{e^{-2t}\sin(3t)}$	
d		$\boldsymbol{\frac{s}{s^2-25}}$
e		$\boldsymbol{\frac{s}{s^2+4}}$
f		$\boldsymbol{\frac{1}{s^2+4}}$
g		$\boldsymbol{e^{-3s}\frac{s}{s^2+4}}$
h		$\boldsymbol{\frac{18}{(s+4)^2+9}}$
i	$\boldsymbol{H(t-2)H(t-3)}$	
j		$\boldsymbol{e^{-3s}\mathcal{L}\left(t^2-1\right)}$
k	$\boldsymbol{\left(t^2-4t+4\right)H(t-2)}$	

2 Solve the initial-value problem consisting of the differential equation

$$\boldsymbol{y' = -\frac{4}{y\sqrt{t}}}$$

along with the initial condition $\boldsymbol{y(1) = -3}$. Show solution steps and obtain an explicit solution for $\boldsymbol{y}$ in terms of $\boldsymbol{t}$.

3 Compute the *convolution* of the functions $\boldsymbol{f(t) = e^{2t}}$ and $\boldsymbol{g(t) = e^{-2t}}$.

4 Use the Reduced-Row-Echelon-Form technique to find the solution(s) of the following system:

$$\begin{aligned} \boldsymbol{x + 2y + 3z} &= \mathbf{9} \\ \boldsymbol{2x - y + z} &= \mathbf{8} \\ \boldsymbol{3x - z} &= \mathbf{3} \end{aligned}$$

5 Find all the eigenvalues of $\boldsymbol{A} = \begin{bmatrix} \mathbf{2} & \mathbf{4} \\ \mathbf{5} & \mathbf{3} \end{bmatrix}$. Find an eigenvector for each of the eigenvalues.

6 Use the Laplace-Transform Technique to solve the initial-value problem $\boldsymbol{y'' + 4y = -8e^{2t}}$ with $\boldsymbol{y(0) = 1}$ and $\boldsymbol{y'(0) = 0}$

7 Show steps in using *The Method of Variation of Parameters* to find a general-solution formula for the differential equation $\boldsymbol{y'' - 4y' + 4y = 6te^{2t}}$

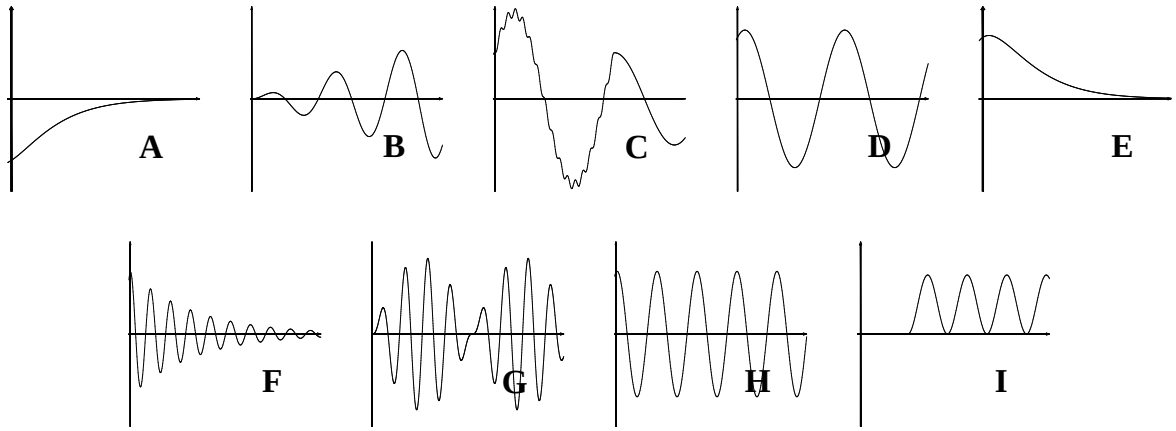
8 Consider the following reduced row-echelon augmented matrices:

$$\boldsymbol{A} = \left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{5} \end{array}\right] \quad \boldsymbol{B} = \left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{2} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right] \quad \boldsymbol{C} = \left[\begin{array}{ccc|c} \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right] \quad \boldsymbol{D} = \left[\begin{array}{cc|c} \mathbf{1} & \mathbf{0} & \mathbf{3} \\ \mathbf{0} & \mathbf{1} & \mathbf{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right]$$

Fill in the table. If the system corresponding to the matrix named in the left column has just one solution, write the solution, as a column vector in the "One solution" column. If the system has many solutions, write **TWO** of them as column vectors in the "Many solutions" column. If the system has no solutions, put a big checkmark in the "No solutions" column.

	One solution	Many solutions	No solutions
A			
B			
C			
D			

9 Consider the following graphs of solutions of initial-value problems:



For each of the following initial-value problems, enter the letter of the best corresponding graph from the above collection (***H*** denotes the Heaviside function):

(a) ____ $\mathbf{2y'' + 2y' + 200y = 0 \quad y(0) = 1, y'(0) = 1}$

(b) ____ $\mathbf{y'' + 4y = 200\big(1 - H(t - \pi)\big)\sin(32t) \quad y(0) = 2, y'(0) = 0}$

(c) ____ $\mathbf{y'' + 25y = 12H(t - 3/2) \quad y(0) = 0, y'(0) = 0}$

(d) ____ $\mathbf{y'' + 7y' + 12y = 0 \quad y(0) = 1, y'(0) = 1}$

(e) ____ $\mathbf{y'' + 7y' + 12y = 0 \quad y(0) = -1, y'(0) = 1}$

(f) ____ $\mathbf{y'' + 25y = 0 \quad y(0) = 1, y'(0) = 1}$

(g) ____ $\mathbf{y'' + 4y = 0 \quad y(0) = 1, y'(0) = 1}$

(h) ____ $\mathbf{y'' + 4y = \sin(1.6t) \quad y(0) = 0, y'(0) = 0}$

(i) ____ $\mathbf{y'' + 9y = \cos(3t) \quad y(0) = 0, y'(0) = 0}$