

This test has pages 1 – 5. Take a moment to make sure you have them all.

No Calculators Allowed; No Reference Materials; Just You and Your Pencil and Eraser.

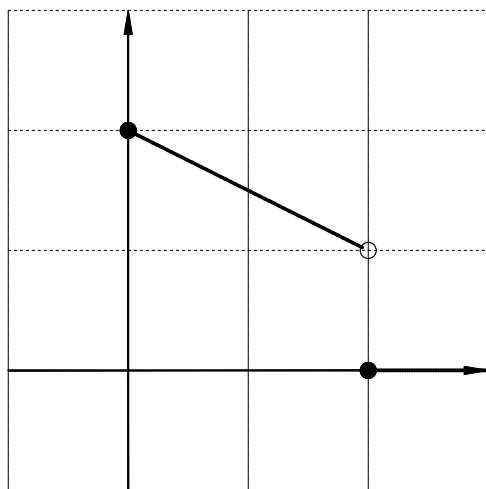
1 Fill out your own Laplace-Transform Cheat Sheet here.

$f(t)$	$F(s)$	$f(t)$	$F(s)$
1		t^n	
e^{At}		$H(t - A)$	
$\cos(At)$		$\sin(At)$	
$f'(t)$		$\sinh(At)$	
$H(t - A)f(t - A)$		$tf(t)$	
	$\frac{1}{s}$		$\frac{1}{s^n}$
	$\frac{1}{s - A}$		$e^{-sA}F(s)$
	$F(s - A)$		$F'(s)$
	$\frac{12}{(s + 1)^2 + 9}$		$\frac{7s}{(s + 1)^2 + 9}$

2 State the basic definition of the Laplace Transform of f .

- 3 Let $F(s)$ be the Laplace Transform of $f(t)$, where $F(s) = \frac{6s^2 + s - 6}{s^2(s + 2)}$. Find $f(t)$.

- 4 Find the Laplace transform of the function g whose graph is shown. The coordinate squares are one unit on a side, and the graph of g coincides with the x -axis for $x \in (2, \infty)$.



- 5 Show steps in using the Laplace-Transform method to solve the initial-value problem:

$$\mathbf{y}' = -2\mathbf{y} + e^{2t} \qquad \mathbf{y}(0) = 3/4$$

- 6 Let $\mathbf{g}(t) = e^{At}\mathbf{f}(t)$. State and prove the relation between $\mathbf{F}(s)$ and $\mathbf{G}(s)$.

7 Use the Laplace-Transform technique to solve the initial-value problem

$$y' + 2y = E(t) \quad y(0) = 0,$$

where $E(t) = \begin{cases} 0 & \text{if } 0 \leq t < 2 \\ 4 & \text{if } t \geq 2 \end{cases} .$

Give an answer in form $y(t) = \begin{cases} \underline{\hspace{4cm}} & \text{if } 0 < t < 2 \\ \underline{\hspace{4cm}} & \text{if } t \geq 2 \end{cases}$

8 Here’s a list of second-order linear differential equations.

(A) $2y'' + 2y' + 81y = 0$

(B) $2y'' - 2y' + 81 = 0$

(C) $y'' - 4y = 0$

(D) $y'' + 4y = 0$

(E) $y'' + 4y = \cos(2.25t)$

(F) $y'' + 4y = \cos(2t)$

Each of the figures below shows a portion of a *composite plot* of a solution of one of the above differential equations. This means that we have plotted the parametric equations

- (i) $x = y(t)$, where $y(t)$ is the solution,
- (ii) $y = v(t)$, where $v = y'$, and
- (iii) $z = t$, on the vertical as shown.

Match graph with differential equation by entering the letter of the appropriate differential equation from the above list in the box in the lower-right part of each figure.

