Thu Sep 16 19:21:53 MDT 2004 /m333.fa04/handouts333/t1_333_917/t1_333_917

This test has pages 1 – 5. Take a moment to make sure you have them all.

No Calculators Allowed; No Reference Materials; Just You and Your Pencil and Eraser.

1 Solve the initial-value problem: $\boldsymbol{y' = 2x(y+1)^2}$ with $\boldsymbol{y(2) = 0}$.

2 Solve the initial-value problem: $\boldsymbol{y' - 2xy = 2xe^{x^2}}$ with $\boldsymbol{y(0) = 8}$.

3 Draw a direction field for the differential equation $\boldsymbol{y' = -2x + y - 2}$. It is enough that you indicate the "bewhiskered" isoclines for $\boldsymbol{m = 0}$, $\boldsymbol{m = 1}$, and $\boldsymbol{m = -1}$ all on one graph. Indicate the approximate shape of the solution of the initial-value problem

$$\boldsymbol{y' = -2x + y - 2 \qquad y(-1) = 0.}$$

Do *not* solve this initial-value problem. Just indicate the shape of its graph.

4 Quote the Existence and Uniqueness Theorem for First-Order Linear differential equations. Be sure to mention the guarantee for the interval of existence.

5 Consider the initial-value problem $(x^2 - 4)y' + 3xy = 6x$ with $y(-1) = 2$. What interval of existence is guaranteed for the solution of this initial-value problem? Briefly, why?

6 Here are some famous differential equations:

(1) $\dfrac{dy}{dx} = 2y$ (2) $\dfrac{dy}{dx} = -2y$ (3) $\dfrac{dy}{dx} = 2x$ (4) $\dfrac{dy}{dx} = -2x$

(i) As is well known, each of these differential equations has infinitely many solutions.

(ii) Each of the plots below shows a solution of one of the above differential equations.

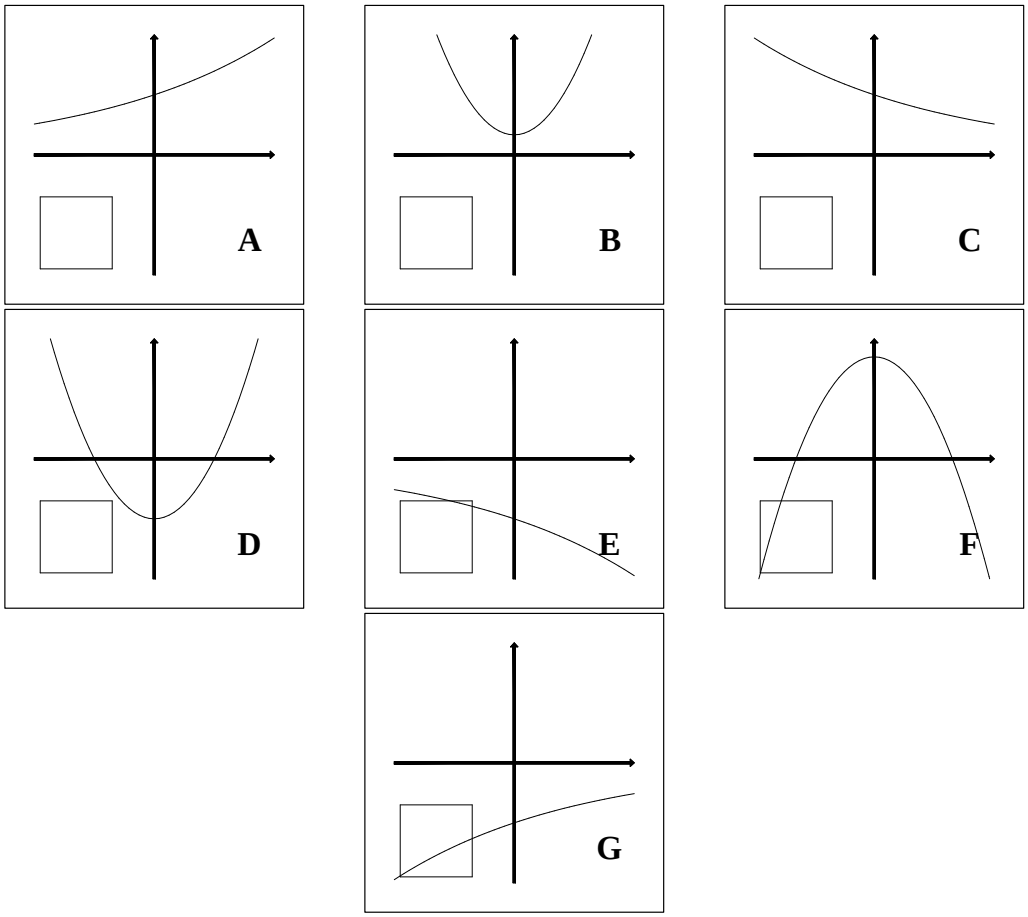
(iii) Each of the plots below has a square box in its lower-left corner.

(iv) For each plot, fill in the square box with the number of the appropriate differential equation from above.

(v) Each box has only ONE correct entry.

(vi) But some of the differential equations' numbers will appear in more than one box.

(vii) No box should be left empty.



7 A 500-gallon tank contains 400 gallons of a solution containing $\mathbf{100}\,\boldsymbol{lbs}$ of salt. Solution containing $\mathbf{5}\,\boldsymbol{lbs}$ of salt in each gallon starts running into the tank at $\mathbf{4}\,\boldsymbol{gal/min}$. While this is going on, large paddles stir up the solution in the tank, and solution is pumped out at $\mathbf{2}\,\boldsymbol{gal/min}$. Find a formula for the amount ($\boldsymbol{lbs}$) of salt in the tank after $\boldsymbol{t}$ minutes.