Final form: Thu Apr 26 17:24:15 MDT 2007 /m301.sp07/handouts301/t3_301_427/review_suggestions_3_tex

1 This list is now in final form. Makes sure you scan it all for red stuff.

2 Test #3 is

Friday
4/27/07.

3 The test will cover the material of Assignments #14 – #18. roughly, that is, sections 3.6, 3.7, 4.1, 4.2, 4.4, 4.5, 4.6, 4.7.

4 Exam ground rules:

(a) Watch out for verbiage along these lines:

> This problem asks you to prove a special case of a general theorem. Your solution to the problem must be "from first principles", that is, it must not depend upon the more general theorem, or upon any of its corollaries.

(b) This is a no-calculators exam.

(c) After the test, take along your question sheets. From the problems you attempted, you may select two to resubmit on Monday. For each of these problems, this means that:

- (a) If you got a perfect score on the in-class version of the problem, you get no additional points for the problem.
- (b) If you do better on the in-class version than on the resubmission, there's no penalty.
- (c) If the resubmission is better than the in-class version, the in-class version's score will be boosted by 90% of the difference between the scores.

5 Things to know about:

(i) Generalizing the dot product to higher-dimensional spaces.

(ii) Generalizing the idea of projection of one vector onto another to higher dimensions.

(iii) How to change a basis into an *orthogonal* basis.

(iv) How to find coordinates relative to any basis.

(v) How to find coordinates relative to an *orthonormal* basis.

(vi) Linear transformation.

(vii) Standard-basis matrix of a linear transformation.

(viii) How to derive things like the famous *polarization identity,* which figured in our proof that a length-preserving (orthogonal) linear transformation also preserves dot products.

(ix) How is it that the columns of the standard-basis matrix of an orthogonal linear transformation turn out to be an orthonormal set.

(x) The inverse of the standard-basis matrix of an orthogonal linear transformation (turns up later in the diagonalization of a symmetric matrix).

(xi) In $\mathbb{R}^2$, the determinant of the standard-basis matrix of an orthogonal linear transformation.

(xii) The linear transformations with the standard-basis matrices

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \cos(2\beta) & \sin(2\beta) \\ \sin(2\beta) & -\cos(2\beta) \end{bmatrix}$$

6 Determinant stuff to be up on:

(i) Laplace's Rule for reducing an $n \times n$ determinant to a "linear combination" of $(n-1) \times (n-1)$ determinants.

(ii) Eyeballing determinants: in some situations you can shortcut the determinant. Triangular and diagonal matrices, for example.

(iii) Connection of determinants to

(a) nonsingularity

(b) row equivalence

(c) similarity

7 Eigenstuff to be up on:

(i) The formal definition of *eigenvalue* and of *eigenvector* for a square matrix A.

(ii) How the *characteristic polynomial* arises in this context.

(iii) The *eigenspace* associated with a particular eigenvalue is the null space of which matrix?

(iv) Eyeballing eigenvectors.

(v) Can you state and prove the theorem about linear independence of eigenvectors corresponding to different eigenvalues.

8 Find an example in support of the idea that row equivalence and similarity are different. Maybe, for instance, a pair of row-equivalent matrices which aren't similar. Or, maybe, a proof that row equivalence and similarity *are so* the same thing.

9 They say that the plane $\boldsymbol{ax + by + cz = 0}$ is a subspace of $\mathbb{R}^{\mathbf{3}}$. A basis for this subspace? An *orthonormal* basis?

10 Refer to problem 4(b), Test #2: what if we change $\boldsymbol{W}$ to the set of all *symmetric* matrices in $\boldsymbol{\mathcal{M}}$?