Final Form: Wed Mar 21 18:04:12 MST 2007

1. This list is now in final form.

2. Test #2 is

   Friday
   3/23/07.

3. The test will cover the material of Assignments #8 – #14. roughly, that is, sections 1.9, 2.1-2.4, 3.1-3.5.

4. This is a no-calculators exam.

5. **Definitions and Questions About Definitions**

   (a) Dependence equation

   (b) Nonsingular matrix

   (c) Properties equivalent to nonsingularity

   (d) Geometric vectors and pictures of their sums, differences, and scalar multiples.

   (e) The algebraic parallelism criterion.

   (f) The polar form of a vector in $\mathbb{R}^2$.

   (g) The dot product in $\mathbb{R}^2$ and $\mathbb{R}^3$, its relation to vector length, and angle.

   (h) The algebraic perpendicularity criterion.

   (i) The projection of one vector onto another, both pictorially and algebraically.

   (j) The vector-parallel equation of a line in both $\mathbb{R}^2$ and $\mathbb{R}^3$.

   (k) The vector-normal equation of a line in $\mathbb{R}^2$.

   (l) The vector-normal equation of a plane in $\mathbb{R}^3$.

   (m) The properties of an abstract vector space.

   (n) The definition of *subspace*.

   (o) The definition of the subspace spanned by a set of vectors.

   (p) Nullspace of a matrix
(q) Range of a matrix.
(r) Column space for a matrix
(s) Basis for a subspace
(t) What was the technical lemma that helped to prove that “dimension” of a subspace makes sense.
(u) Dimension of a subspace.
(v) The cross product of two vectors in $\mathbb{R}^3$:
   (i) facile computation
   (ii) commutativity
   (iii) associativity
   (iv) orthogonality
   (v) relation to area
(w) Intersections in $\mathbb{R}^3$: plane-plane, line-plane, line-line. Sometimes one can determine whether two objects intersect without actually solving for the intersection point(s).

6 Computations

(a) In $\mathbb{R}^2$ straight-line equations
   (i) vector-parallel form
   (ii) vector-normal form
(b) Compute the angle between two vectors in $\mathbb{R}^2$. In $\mathbb{R}^3$.
(c) Check whether a set is linearly independent.
(d) Check whether a given vector is in a given span.
(e) Compute bases (and dimensions) for null spaces, ranges, and column spaces.
(f) Compute a basis for a row space.
(g) Expressing objects as spans of simple sets of vectors: $\mathcal{N}(A)$, a plane through the origin...

7 Proofs
I will offer a choice of proofs near to in-class proofs or home-work proofs. I will also attempt to cook up a choice of things you’ve never seen before. Like for instance:
(a) Write a proof that \( S \subseteq T \).

(b) Prove that a given set is a subspace.

(c) The elementary homework properties of an abstract vector space: uniqueness of \( \bar{0} \), uniqueness of additive inverse, \( 0\bar{v} \)

(d)

(e)

The above is not an exhaustive list.