1 Let

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{5} & \mathbf{3} \\ \mathbf{8} & \mathbf{5} \end{bmatrix}$$

Find, if possible, an *inverse from the left* for $\boldsymbol{A}$. That is, find a matrix $\boldsymbol{L}$ such that $\boldsymbol{L}\,\boldsymbol{A} = \boldsymbol{I}$. If it is not possible to find such an $\boldsymbol{L}$, explain briefly. If it *is* possible to find such an $\boldsymbol{L}$, explain whether it is unique.

It is certainly true that a nonsquare matrix such as $\boldsymbol{A}$ cannot have an inverse, but $\boldsymbol{L}$ is not an inverse, it's an inverse from the left.

2 Let $\boldsymbol{V}$ be the set of all $\mathbf{2} \times \mathbf{2}$ matrices. Let $+$ denote the usual addition of matrices and $*$ the usual multiplication of a matrix by a scalar. Then $\{\boldsymbol{V},\ +,\ *\}$ can be shown to be an Abstract Vector Space (page 168).

You do not have to use the symbol $*$ in the following.

Let

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{3} \\ \mathbf{0} & \mathbf{2} \end{bmatrix}.$$

Determine whether the set

$$\boldsymbol{\mathcal{P}} = \{\boldsymbol{I},\ \boldsymbol{A},\ \boldsymbol{A}^{\mathbf{2}}\}$$

is linearly independent.

3 Given the lines

(A) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$$

(B) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 3 \end{bmatrix} + t \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$$

(C) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

there are three ways to pair them up.

For each pair, check into

(a) Do the two lines coincide?

(b) Are they parallel, but noncoincident?

(c) Are they skew?

(d) Do they cross in a single point?

- (i) If so, give an equation for the plane they form.
- (ii) If so, give the angle at which they meet.