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In class on Tuesday, 3/13/07, we worked on writing Establishment-approved and fashionable proofs of assertions of form $\boldsymbol{S} \subset \boldsymbol{T}$.

Suppose that sets $\boldsymbol{S}$ and $\boldsymbol{T}$ have some specification:

$$\boldsymbol{S} = \left\{\boldsymbol{x} \ : \ \boldsymbol{P(x)} \text{ is true.}\right\} \qquad \text{and} \qquad \boldsymbol{T} = \left\{\boldsymbol{x} \ : \ \boldsymbol{Q(x)} \text{ is true.}\right\}.$$

A proof that $\boldsymbol{S} \subset \boldsymbol{T}$ would go like this:

> Let $\boldsymbol{z} \in \boldsymbol{S}$.
>
> Then $\boldsymbol{P(z)}$ must be true.

There then ensues reasoning showing that, consequently, $\boldsymbol{Q(z)}$ must be true. The proof would continue:

> Hence $\boldsymbol{Q(z)}$ is true.
>
> Hence $\boldsymbol{z} \in \boldsymbol{T}$.
>
> Since $\boldsymbol{z}$ is an arbitrary element of $\boldsymbol{S}$, we have proved that any element of $\boldsymbol{S}$ is also an element of $\boldsymbol{T}$.
>
> Thus $\boldsymbol{S} \subset \boldsymbol{T}$.

In class group work, we proved

> **Theorem A:** If $\boldsymbol{U}$ and $\boldsymbol{V}$ are subspaces of the same abstract vector space, then
>
> $$\boldsymbol{U} + \boldsymbol{V} \subset \boldsymbol{Sp}(\boldsymbol{U} \cup \boldsymbol{V}).$$

1 Write a fashionable, establishment endorseable, proof of

> **Theorem B:** If $\boldsymbol{U}$ and $\boldsymbol{V}$ are subspaces of the same abstract vector space, then
>
> $$\boldsymbol{Sp}(\boldsymbol{U} \cup \boldsymbol{V}) \subset \boldsymbol{U} + \boldsymbol{V}.$$