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This test has pages 1 – 2. Take a moment to make sure you have them all.

No Calculators Allowed; No Reference Materials; Just You and Your Pencil and Eraser.

1 Let $\vec{a} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -8 \\ 1 \\ 4 \end{bmatrix}$.

(a) Compute $\vec{p}$, the projection of $\vec{b}$ onto $\vec{a}$.

(b) Give a simplified expression for the angle between $\vec{b}$ and $\vec{a}$. That is, give an expression which, keyed into a calculator, would give a very close approximation to the angle.

(c) Give a vector equation for the line parallel to $\vec{b}$ which passes through the point whose position vector is $\vec{a}$. Explain whether $(7, 3, 1)$ lies on this line.

(d) Find an equation for the plane through the origin which lies parallel to both $\vec{a}$ and $\vec{b}$.

(e) Explain whether $\begin{bmatrix} 5 \\ 5 \\ -7 \end{bmatrix}$ lies in the subspace spanned by $\vec{a}$ and $\vec{b}$.

2 Let $A = \begin{bmatrix} 2 & 6 & 4 & 2 \\ 1 & 3 & 2 & 1 \\ 3 & 10 & 7 & 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 5 \\ 5 \\ -7 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 6 \\ 3 \\ -6 \end{bmatrix}$.

(a) Explain whether $\vec{v}$ lies in the *range* of matrix A.

(b) Explain whether $\vec{w}$ lies in the *range* of matrix A.

(c) Express each of the vectors $\vec{v}$ and $\vec{w}$ as a linear combination of columns of A.

3 Certainly the *row space* of the matrix A in problem 2 is spanned by three vectors. Explain whether the row space can be spanned by fewer than three vectors.

4 Let the matrix A and the vectors $\vec{v}$ and $\vec{w}$ be as in problem 2. For each of the following sets, explain whether the set is a subspace of R^n for $n =$?

(a) The set of all $\vec{x}$ such that $A\vec{x} = \vec{v}$.

(b) The set of all $\vec{x}$ such that $A\vec{x} = \vec{w}$.

5 Work at least **one** of the following parts (A)-(B). Raise your hand if you need more paper.

(A) Let $\boldsymbol{W}$ be a subspace of $\boldsymbol{R^3}$. Briefly explain what is meant by "$\boldsymbol{W}$ is closed under cross-product operation". Then prove this assertion.

(B) Let $\boldsymbol{W}$ be a subspace of $\boldsymbol{R^3}$. Briefly explain what is meant by the assertion "$\boldsymbol{W}$ is closed under the operation of projecting one $\boldsymbol{W}$ vector onto another $\boldsymbol{W}$ vector". Then prove this assertion.

6 Work at least **one** of the following parts (A)-(B). Raise your hand if you need more paper.

(A) Let $\boldsymbol{A}$ be an $\boldsymbol{m}$-by-$\boldsymbol{n}$ matrix. Suppose $\left\{\vec{\boldsymbol{v}}_1, \vec{\boldsymbol{v}}_2, \ldots, \vec{\boldsymbol{v}}_q\right\}$ is a set of vectors in $\boldsymbol{R^n}$ such that $\left\{\boldsymbol{A}\vec{\boldsymbol{v}}_1, \boldsymbol{A}\vec{\boldsymbol{v}}_2, \ldots, \boldsymbol{A}\vec{\boldsymbol{v}}_q\right\}$ is a linearly-independent set of vectors in $\boldsymbol{R^m}$. Prove that $\left\{\vec{\boldsymbol{v}}_1, \vec{\boldsymbol{v}}_2, \ldots, \vec{\boldsymbol{v}}_q\right\}$ is a linearly-independent set.

(B) If $\boldsymbol{A}$ and $\boldsymbol{B}$ are subsets of $\boldsymbol{R^n}$, then the set $\boldsymbol{A} + \boldsymbol{B}$ is defined to be the set of all possible sums of form $\vec{\boldsymbol{a}} + \vec{\boldsymbol{b}}$, where $\vec{\boldsymbol{a}} \in \boldsymbol{A}$ and $\vec{\boldsymbol{b}} \in \boldsymbol{B}$.

Let $\boldsymbol{U}$ and $\boldsymbol{V}$ be subspaces of $\boldsymbol{R^n}$. Prove that the set $\boldsymbol{U} + \boldsymbol{V}$ is also a subspace of $\boldsymbol{R^n}$.