Consider the matrix

$$\boldsymbol{S} = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{3} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{4} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{5} \end{bmatrix}.$$

1 Since $\boldsymbol{S}$ is upper triangular, we know that the upper-left-most entry, $\boldsymbol{s_{11}} = \mathbf{2},$ is an eigenvalue, $\boldsymbol{\lambda} = \mathbf{2},$ with eigenvector $\vec{\boldsymbol{e}}_{\mathbf{1}}.$ Could there be other eigenvectors for $\boldsymbol{\lambda} = \mathbf{2}$ independent of $\vec{\boldsymbol{e}}_{\mathbf{1}}$?

2 Let's move on to find eigenvectors for eigenvalue $\boldsymbol{\lambda} = \mathbf{3}.$ In the space below from left to right, write in $(\boldsymbol{S} - \mathbf{3}\boldsymbol{I}),$ and then an RREF of $(\boldsymbol{S} - \mathbf{3}\boldsymbol{I}),$ and *then* an eigenvector for $\boldsymbol{\lambda} = \mathbf{3}.$

3 Next, to find eigenvectors for eigenvalue $\boldsymbol{\lambda} = \mathbf{4} :$ In the space below from left to right, using a small font, write in $(\boldsymbol{S} - \mathbf{4}\boldsymbol{I}),$ and then an RREF of $(\boldsymbol{S} - \mathbf{4}\boldsymbol{I}),$ and *then* an eigenvector for $\boldsymbol{\lambda} = \mathbf{4}.$

4 Finally, we go after eigenvectors for eigenvalue $\boldsymbol{\lambda = 5}$ **:**

5 Now, let $\boldsymbol{T}$ be the matrix be a new matrix obtained from $\boldsymbol{S}$ by changing the value of $\boldsymbol{s_{22}}$ from **3** to **2**. What's the set of eigenpairs like for this matrix $\boldsymbol{T}$?