These are alleged answers. For each error herein, you get extra-credit points for being the first to report it by e-mail.

1. To evaluate $\int_0^{\pi/6} x \cos(2x) \, dx$, we start with the “parts” transformation,

\[ u = x \quad v' = \cos(2x) \]

\[ u' = 1 \quad v = \frac{\sin(2x)}{2} \]

which yields

\[ \int_0^{\pi/6} x \cos(2x) \, dx = \frac{x \sin(2x)}{2} \bigg|_0^{\pi/6} - \int_0^{\pi/6} \frac{\sin(2x)}{2} \, dx \]

\[ = \frac{\pi \sin(\pi/3)}{12} - \frac{1}{2} \int_0^{\pi/6} \sin((2x)) \, dx \]

\[ = \frac{\pi \sqrt{3}}{24} + \frac{1}{4} \cos(2x) \bigg|_0^{\pi/6} \]

\[ = \frac{\pi \sqrt{3}}{24} + \frac{1}{4} \left( \frac{1}{2} - 1 \right) \]

\[ = \frac{\pi \sqrt{3} - 3}{24} \]
2. Here is the square-of-sine-or-cosine trick:

\[
\cos(\theta)^2 = \frac{1 + \cos(2\theta)}{2} \quad \text{and} \quad \sin(\theta)^2 = \frac{1 - \cos(2\theta)}{2}
\]

This makes our problem go

\[
\int_0^{\pi/12} \cos(3x)^2 \, dx = \int_0^{\pi/12} \frac{1 + \cos(6x)}{2} \, dx
\]

\[
= \frac{1}{2} \int_0^{\pi/12} (1 + \cos(6x)) \, dx
\]

\[
= \frac{1}{2} \left( x + \frac{1}{6} \sin(6x) \right) \bigg|_0^{\pi/12}
\]

\[
= \frac{1}{2} \left( \frac{\pi}{12} + \frac{1}{6} \sin(\pi/2) \right) = \frac{1}{2} \left( \frac{\pi}{12} + \frac{1}{6} \right) = \frac{\pi + 2}{24}
\]