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These are alleged answers. For each error herein, you get extra-credit points for being the first to report it by e-mail.

1 (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{(-5)^n}$: A convergent geometric series.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{(-5)^n} = \sum_{n=2}^{\infty} \left(\frac{1}{5}\right)^n = \frac{1}{25}\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n = \frac{1}{25} \times \frac{1}{1-\frac{1}{5}} = \frac{1}{20}$$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{5^n}$: Converges. One way to see this: note that for $n > 1,$ we have $n < 2^n.$ Thus

$$\frac{n}{5^n} < \frac{2^n}{5^n} = \left(\frac{2}{5}\right)^n.$$

This means that the given series converges by comparison with convergent $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n.$

The ratio test or root test will tell a similar story.

(c) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right)$: Convergent. One way to see this is to combine the k^{th} term so we are looking at $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)},$ which can be seen to converge: $\displaystyle\frac{1}{k(k+1)} < \frac{1}{k^2}.$

Another approach: the partial sum S_N telescopes:

$$S_N = \sum_{k=1}^{N} \left(\frac{1}{k} - \frac{1}{k+1}\right) = 1 - \frac{1}{N+1}.$$

This means that the series converges, because $\displaystyle\lim_{N\to\infty} S_N$ exists, and that its sum is $\displaystyle\lim_{N\to\infty} S_N = 1.$