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These are alleged answers. For each error herein, you get extra-credit points for being the first to report it by e-mail.

1 (a) Since $\mathbf{2+(-1)^n \leqslant 3}$, we have

$$\frac{2+(-1)^n}{n^2} \leqslant \frac{3}{n^2},$$

so we can say that this series converges by comparison with convergent $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2}$.

One could also try the limit comparison test:

$$\frac{\dfrac{2+(-1)^n}{n^2}}{\dfrac{1}{n^{3/2}}} = \frac{2+(-1)^n}{n^{1/2}} \to 0,$$

which again indicates convergence.

(b) Since $\mathbf{2+(-1)^n \geqslant 1}$, we have

$$\frac{2+(-1)^n}{n} \geqslant \frac{1}{n},$$

so we can say that this series diverges by comparison with divergent Harmonic Series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

Probably one can get a limit compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ to indicate divergence for you.

(c) A limit compare does this one nicely:

$$\frac{\dfrac{k-3}{\sqrt[3]{k^8-37}}}{\dfrac{1}{k^{5/3}}} = \left(\frac{k-3}{\sqrt[3]{k^8-37}}\right)\left(k^{5/3}\right) = \frac{k^{8/3}\left(1-\dfrac{3}{k}\right)}{k^{8/3}\sqrt[3]{1-\dfrac{37}{k^8}}}.$$

This last obviously goes to $\mathbf{1}$ as $k \longrightarrow \infty,$ so the given series converges by limit compare with convergent $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{5/3}}$.