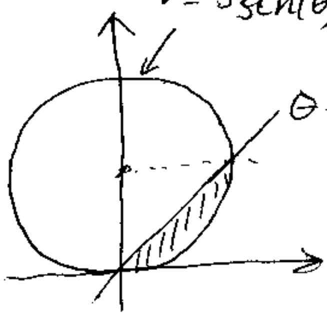
MATH 175 004 – Quiz – 10/27/06 – Name: KEY 1

Fri Oct 27 12:36:42 MDT 2006 /m175.fa06/handouts175/qA27/qA27_175

Pencils and Erasers Only – No Calculators Allowed.

1 Find the exact length of the polar-curve arc

$$r = 3\sin(\theta) \qquad \text{with} \qquad 0 \leqslant \theta \leqslant \pi/4.$$



$$L = \int_0^{\frac{\pi}{4}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Now $\frac{dr}{d\theta} = 3\cos(\theta)$ so

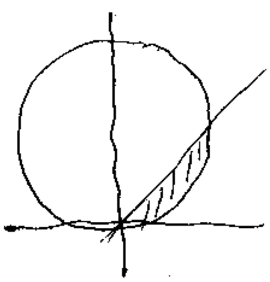
$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = 9\sin(\theta)^2 + 9\cos(\theta)^2 =$$

$$= 9\left(\sin(\theta)^2 + \cos(\theta)^2\right) = 9(1) = 9$$

$$\therefore L = \int_0^{\frac{\pi}{4}} \sqrt{9}\, d\theta = 3\int_0^{\frac{\pi}{4}} d\theta = 3\left(\frac{\pi}{4}\right) = \frac{3\pi}{4}.$$

Check by grade-8 methods: $L = \frac{1}{4}\text{perimeter} = \frac{1}{4}(\pi d) = \frac{3\pi}{4}$

2 Find the exact area of the sector indicated in problem 1. That is, find the area of the Quadrant-I region enclosed by the polar curves $r = 3\sin(\theta)$, $\theta = 0$, and $\theta = \pi/4$.



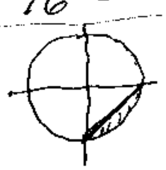
$$A = \frac{1}{2}\int_0^{\frac{\pi}{4}} r^2\, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} 9\sin(\theta)^2\, d\theta$$

$$= \frac{9}{2}\int_0^{\frac{\pi}{4}} \left(\frac{1-\cos(2\theta)}{2}\right) d\theta = \frac{9}{4}\int_0^{\frac{\pi}{4}} \left(1-\cos(2\theta)\right) d\theta$$

$$= \frac{9}{4}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{4}} = \frac{9}{4}\left[\frac{\pi}{4} - \frac{1}{2}\sin\left(\frac{\pi}{2}\right)\right] = \frac{9}{4}\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{9}{16}(\pi - 2).$$

Check



Ans Area = sector Area − Δ area

$$= \frac{1}{4}\left(\pi\left(\frac{3}{2}\right)^2\right) - \frac{1}{2}\left(\frac{3}{2}\right)^2 = \frac{9}{4}\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{9}{4}\left(\frac{\pi - 2}{4}\right) = \underline{\underline{OK}}$$