This test has page 1 - 6. Take a moment to make sure you’ve got them all.
No Calculators Allowed; No Reference Materials; Just You and Your Pencil and Eraser.

1. Find the sums. If there is no such sum, explain briefly:

(a) \[ \sum_{n=0}^{\infty} \frac{2^n}{5^n} \]

(b) \[ \sum_{n=0}^{\infty} \frac{2^n}{n!} \]

(c) \[ \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n \]
2 Show steps in evaluating the following:

(a) \[ \int_0^{\pi/2} \cos(2x)^2 \, dx \]

(b) \[ \int_0^\infty e^{-x/10} \, dx \]
3 Show how to break the integral \[ \int \frac{x + 2}{x^2 + 11x - 26} \, dx \] up into a sum of simpler integrals. Don’t evaluate the simpler integrals.

4 Transform the integral \[ \int_{0}^{2\sqrt{3}} \frac{x^3}{\sqrt{16 - x^2}} \, dx \] by means of an appropriate trigonometric substitution. Show the new simplified integrand and the new limits, but don’t evaluate the new integral.
5. Let $\mathcal{M}$ be the region enclosed by the curves $y = x^2$, $x = 2$, and $y = 6$ with $x \geq 2$.

(a) Set up, but do not evaluate, an integral whose value is the length of the curved portion of the boundary of $\mathcal{M}$.

(b) Set up, but do not evaluate, an integral whose value is the volume of the region swept out when $\mathcal{M}$ is revolved about the line $x = 3$. 
6 Let $S$ denote the infinite series
\[ \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{10^n n}. \]

(a) Show steps in determining the interval of convergence of $S$.

(b) Approximate the sum of $S$ when $x = 1/100$ to within $\frac{1}{30,000,000}$. Be sure to explain how long a partial sum is needed. Write your approximation as a single quotient of integers.
7 Most of the graphs below show the first four partial sums of the Taylor series for some $f$ about some $x = a$. The highest-degree partial sum is the darkest graph. Enter the letter of the appropriate graph in the appropriate blank below. If some blank has no corresponding graph, enter an "X":

\(\text{(i) } \cos(x), \ a = \frac{\pi}{3}\) \hspace{1cm} \(\text{(iv) } f(x) = e^x, \ a = 1\)

\(\text{(ii) } f(x) = \frac{2}{3}x + 1, \ a = 3\) \hspace{1cm} \(\text{(v) } f(x) = e^{-x}, \ a = 1\)

\(\text{(iii) } \sin(x), \ a = \frac{\pi}{6}\) \hspace{1cm} \(\text{(vi) } \cos(x), \ a = \pi\)