This with-calculator portion of the test consists of just three problems.

1. Consider the series \( \sum_{n=0}^{\infty} \frac{(-1)^n}{100^n n!} \)

   (a) Explain why this series has a value.

   (b) Find an approximation to the value of this series which is guaranteed to be within 0.001 of exact value.

   Be sure you explain enough that your work can be replicated exactly from what you write.
2 Define sequence $c_n$ as follows:

\[
\begin{align*}
    c_1 &= 2 \text{ radians} \\
    c_{n+1} &= \cos(c_n)
\end{align*}
\]

Give approximations to the values of the following:

(a) $\lim_{n \to \infty} c_n$

(b) $\sum_{n=1}^{\infty} c_n$

Please turn over!
3. Show steps in in determining the convergence behavior of the following. Give a brief prose explanation as well.

(a) \( \lim_{n \to \infty} \left( 1 - \frac{1}{n} \right)^{n^2} \)

(b) \( \sum_{k=0}^{\infty} \frac{1}{k!} \)

(c) \( \sum_{n=2}^{\infty} \frac{1}{n^2 + (-1)^n} \)

(d) \( \sum_{k=2}^{\infty} (-5)^{-k} \)
4 Show steps in in determining the convergence behavior of the following. Give a brief prose explanation as well.

(a) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} \)

(b) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+3}}{\sqrt[3]{n}} \)

(c) \( \sum_{k=1}^{\infty} \frac{k^k}{k!} \)
5. Show steps in in determining the convergence behavior of the following. Give a brief prose explanation as well.

(a) \[ \sum_{n=1}^{\infty} \frac{n!}{3 \cdot 6 \cdot 9 \cdot \cdots \cdot (3n)} \]

(b) \[ \sum_{n=1}^{\infty} \frac{(3n)!}{3 \cdot 6 \cdot 9 \cdot \cdots \cdot (3n)} \]
6. Show steps in finding the $N$-recipe for the limit $\lim_{n \to \infty} \frac{3n^2 - 11}{2n^2 - 7}$. Be sure to state clearly how $N$ is to be chosen.