

**This with-calculator portion of the test consists of just two problems.**

1 Consider the series  $\sum_{n=0}^{\infty} \frac{(-1)^n}{100^n n!}$

(a) Explain why this series has a value.

(b) Find an approximation to the value of this series which is guaranteed to be within **0.001** of exact value.

Be sure you explain enough that your work can be replicated exactly from what you write.

2 Define sequence  $c_n$  as follows:

$$\begin{aligned}c_1 &= 2 \text{ radians} \\c_{n+1} &= \cos(c_n)\end{aligned}$$

Give approximations to the values of the following:

(a)  $\lim_{n \rightarrow \infty} c_n$

(b)  $\sum_{n=1}^{\infty} c_n$

This non-calculator portion of the test has pages 3 – 6. Take a moment to make sure you have them all.

No Calculators Allowed; No Reference Materials; Just You and Your Pencil and Eraser. Show your steps.

- 3 Show steps in determining the convergence behavior of the following. Give a brief prose explanation as well.

(a)  $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{n^2}$

(b)  $\sum_{k=0}^{\infty} \frac{1}{k!}$

(c)  $\sum_{n=2}^{\infty} \frac{1}{n^2 + (-1)^n}$

(d)  $\sum_{k=2}^{\infty} (-5)^{-k}$

- 4 Show steps in determining the convergence behavior of the following. Give a brief prose explanation as well.

(a) 
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+3}}{\sqrt[3]{n}}$$

(c) 
$$\sum_{k=1}^{\infty} \frac{k^k}{k!}$$

- 5 Show steps in determining the convergence behavior of the following. Give a brief prose explanation as well.

(a) 
$$\sum_{n=1}^{\infty} \frac{n!}{3 \cdot 6 \cdot 9 \cdot \dots \cdot (3n)}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{(3n)!}{3 \cdot 6 \cdot 9 \cdot \dots \cdot (3n)}$$

- 6 Show steps in finding the  $N$ -recipe for the limit  $\lim_{n \rightarrow \infty} \frac{3n^2 - 11}{2n^2 - 7}$ . Be sure to state clearly how  $N$  is to be chosen.