1. Evaluate the integrals. Simplify your answers:

(a) \( \int_0^{\pi/2} x \cos(x) \, dx \)

(b) \( \int_0^{\pi/2} \cos(x)^3 \, dx \)

(c) \( \int_0^4 \frac{1}{x^2 - 64} \, dx \)
2. Give the initial partial-fractions guess for the rational expression

\[ \frac{3x^2 - 7x + 2}{x(x^2 - 9)^2(x^2 + 9)^2} \]

3. Nerpel made the trigonometric substitution \( x = 5 \tan(\theta) \) in an antiderivative problem. He went on to compute an antiderivative:

\[ Q = \theta - \tan(\theta) + 3 \sin(\theta) + \sin(2\theta) + C \]

Get this antiderivative into final form, that is, in terms of \( x \). Your final answer must minimize the number of trig-function appearances.
4. Let \( J = \int_{0}^{\sqrt{3}} \frac{x^2}{\sqrt{4 - x^2}} \, dx \)

(a) An appropriate trigonometric substitution for \( J \) is \( x = \) ________ (use \( \theta \) for the new variable).

(b) In terms of \( x \), \( \theta = \) ________

(c) The new \( d\theta \) version of \( J \) will be an integral

\[
\text{from } \theta = \text{__________} \text{ to } \theta = \text{__________}
\]

(d) Accordingly, the new \( d\theta \) version of \( J \) is:

(e) Show steps in evaluating this new \( d\theta \) version of \( J \).
In this problem, you are to show steps in setting up an integral which gives the work necessary to pump out the tank in the following situation. Don’t evaluate the integral – just set it up in a form that shows you know what’s what.

A spherical tank, 10 feet in diameter, contains water to a depth of 7 feet. One can’t put more water in the tank because someone has put an overflow aperture right at the current water level.

How much work does it take to pump the tank out through this overflow?
6. An infinite set of weights is distributed along the $y$-axis as follows:

for each $n = 0, 1, 2, \ldots$ we have $\frac{2^n}{3^{n+2}}$ pounds at the point $(0, -2.5^n)$

This means that there’s $\frac{1}{9}$ pound at $(0, -1)$, $\frac{2}{27}$ pound at $(0, -2.5)$ and so on. We assume that the force of gravity is constant all along the infinite length of the $y$-axis.

For each of the following, either give the finite numerical answer, or else, explain briefly why no such answer exists.

(a) What is the total weight of the infinite set of weights?

(b) How much work must be done to move the entire collection of weights up to the origin?