This with-calculator portion of the test consists of just three problems.

1 Let $\boldsymbol{M_n}$ denote the $\boldsymbol{n}$-subdivision midpoint-sum approximation to $\boldsymbol{J}$, given by

$$\boldsymbol{J = \int_2^4 x^3\,dx.}$$

Compute $\boldsymbol{M_4}$ using your calculator. Show enough work that your solution can be replicated. Show that you know what $\boldsymbol{M_4}$ *is.*

2 We continue from problem 1 with $J = \int_2^4 x^3\,dx$. We know that an upper bound on the error $|J - M_n|$ is given by

$$\frac{K(b-a)^3}{24n^2},$$

where a and b are the limits of integration in J and K is an upper bound on $\left|f''(x)\right|$. Determine how large n must be in order that *this error bound* guarantees M_n to be within $1/10000$ of the true value of J.

3 To four decimal places, $\sqrt{\pi + \pi^{2/3}} \approx$ ________

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This non-calculator portion of the test has pages 3 – 6. Take a moment to make sure you have them all.

No Calculators Allowed; No Reference Materials; Just You and Your Pencil and Eraser. Show your steps.

4 Find the antiderivative: $\displaystyle\int \frac{\boldsymbol{x-1}}{\boldsymbol{x^2-4x-5}}\,\boldsymbol{dx}$

5 Give the partial fractions guess for the case where the denominator of the integrand is $\boldsymbol{(x-8)^3(x^2-4)(x^2+4)}$ and the numerator is a polynomial of degree at most $\mathbf{3}$.

6 Show steps in finding the limits, if any, of the sequences:

(a) $\boldsymbol{a_n = \sqrt{4n^2 + 2n} - 2n}$

(b) $\boldsymbol{a_n = \left(\frac{n+3}{n}\right)^n}$

(c) $\boldsymbol{a_n = \frac{5n+2}{7n+(-1)^n}}$

7 Show steps in evaluating the improper integral: $\displaystyle\int_1^\infty xe^{-2x}\,dx$

8 Smith says that if $f(x)$ is positive for $x \geqslant 1$ and if $\displaystyle\lim_{x\longrightarrow\infty} f(x) = 0$, then $\displaystyle\int_1^\infty f(x)\,dx$ is convergent. Has Smith got it right? Explain briefly.

9 Write down the formal definition of $\lim_{n\to\infty} a_n = L$ for the case where $-\infty < L < \infty$.

10 Determine the convergence behavior of $\displaystyle\int_1^{\infty} \frac{x^2}{\sqrt{1+x^{10}}}\,dx$