This with-calculator portion of the test consists of just three problems.

1. Let $M_n$ denote the $n$-subdivision midpoint-sum approximation to $J$, given by

   $$J = \int_{2}^{4} x^3 \, dx.$$

   Compute $M_4$ using your calculator. Show enough work that your solution can be replicated. Show that you know what $M_4$ is.
2 We continue from problem 1 with \( J = \int_{2}^{4} x^3 \, dx \). We know that an upper bound on the error \( |J - M_n| \) is given by
\[
\frac{K(b - a)^3}{24n^2},
\]
where \( a \) and \( b \) are the limits of integration in \( J \) and \( K \) is an upper bound on \( |f''(x)| \). Determine how large \( n \) must be in order that this error bound guarantees \( M_n \) to be within \( 1/10000 \) of the true value of \( J \).

3 To four decimal places, \( \sqrt{\pi + \pi^{2/3}} \approx \) ________
4 Find the antiderivative: \[ \int \frac{x - 1}{x^2 - 4x - 5} \, dx \]

5 Give the partial fractions guess for the case where the denominator of the integrand is \((x - 8)^3(x^2 - 4)(x^2 + 4)\) and the numerator is a polynomial of degree at most 3.
6. Show steps in finding the limits, if any, of the sequences:

(a) \( a_n = \sqrt{4n^2 + 2n} - 2n \)

(b) \( a_n = \left( \frac{n + 3}{n} \right)^n \)

(c) \( a_n = \frac{5n + 2}{7n + (-1)^n} \)
7 Show steps in evaluating the improper integral: \[ \int_1^\infty xe^{-2x} \, dx \]

8 Smith says that if \( f(x) \) is positive for \( x \geq 1 \) and if \( \lim_{x \to \infty} f(x) = 0 \), then \( \int_1^\infty f(x) \, dx \) is convergent. Has Smith got it right? Explain briefly.
9. Write down the formal definition of \( \lim_{n \to \infty} a_n = L \) for the case where \(-\infty < L < \infty\).

10. Determine the convergence behavior of \( \int_{1}^{\infty} \frac{x^2}{\sqrt{1 + x^{10}}} \, dx \).