Sun Sep 25 16:18:27 MDT 2005 /m171.fa05/handouts171/06_ASSIGNMENT_4_due929/assgn04_925_171

The assignment is to work through part of section 2.4 of our text, and work through the following notes, which extend the discussion of the text a bit.

The part to turn in is described at the end of this sheet.

This sheet contains links.

1 Study the `implicitplot` Command to Graph Two-Variable Equations

1.1 Section 2.4 to the End of the NOTE on Page 31

For this assignment we want to look at just the part concerned with graphing two-variable equations. I want you to pick up on the following:

(a) Entering an equation in Maple. This differs from entering a function or expression.

(b) `with(plots)`, or `with(plots,implicitplot)`. What's the difference between these? What will happen if we issue an `implicitplot` command without prior issue of `with(plots)`, or `with(plots,implicitplot)`?

(c) The minimal `implicitplot` has three operands: the equation, the range for the first variable, and the range for the second variable.

(d) Sometimes Maple uses different unit lengths on the vertical and horizontal axes. This is analogous to `ZoomFit` on TI graphing calculators. Often this is not a bother. But, if your line slopes look wrong, or your circles plot as ovals, and it's distressing, then you need to insert the `scaling` option into whatever plot command you're using:

```
implicitplot(circle, x=XRG, y=YRG, scaling=constrained);
```

This also works in the `plot` command.

(e) At the top of page 31, our text shows an `implicitplot` command to plot two equations. Note that they use the Maple *set* delimiters: `{circle, line}`.

If, rather, you use the Maple *list* delimiters, `[circle, line]`, you can use various plot options to, for instance, make the two curves show up in different colors of your choice. See the example in section 2 below.

1.2 Using `subs` to Modify an Equation and its Graph

For the following, we begin with the radius-**2** circle of the text:

```
circle := x^2 + y^2 = 4;
```

To simplify our eventual `implicitplot` commands, we set up the ranges in a sequence variable:

(i) `XRG := -6..6;  YRG := XRG;`

(ii) `RANGExy := x=XRG, y=YRG;`

1.2.1 Translations

Transform the `circle` equation by replacing all $\boldsymbol{y}$ occurences by $\boldsymbol{(y+1)}$:

```
circle3 := subs(y=y+1, circle);
```

Note that this `subs` command leaves unchanged the original `circle` equation. Try to predict what the following `implicitplot` command will do before you execute it.

```
implicitplot([circle,circle3], RANGExy, colour=[red,black])
```

1.2.2 Dilations and Contractions

Make a new equation `circleA` by replacing all occurences of $\boldsymbol{x}$ by $\boldsymbol{3x}$:

```
circleA := subs(x=3*x, circle);
```

Also

```
circleB := subs(x=x/2, circle);
```

The command

```
implicitplot([circle,circleA,circleB], RANGExy, colour=[red,black,cyan]);
```

was disappointing when I issued it:

(a) the red circle came out as an oval and

(b) one of the curves had undoubtedly incorrect sharp corners on it.

But the `scaling` and `grid=[75,75]` (mid page 30) options fixed it up.

2 Previous Encounters with "Implicit"

Recall *implicit differentiation* from your first-term calculus class. The idea there was to take an equation, such as

$$x^3 + y^3 = 6xy - 3,$$

and, without solving it for y, find an expression for the derivative of y as a function of x:

$$\frac{dy}{dx} = \frac{x^2 - 2y}{2x - y^2}.$$

This last expression could be used to find the slope of the tangent line at a point, such as $(2, 1)$ of the graph of the equation.

You can use Maple to do this

(i) `CUBIC := x^3 + y^3 = 6*x*y - 3;`

(ii) `yPRIME := implicitdiff(CUBIC,y,x);`

(iii) `yp := unapply(yPRIME,x,y);`

(iv) `TanSlope := yp(1,2);`

(v) `TanLineEqn := y-2 = TanSlope*(x-1);`

(vi) `implicitplot([CUBIC,TanLineEqn],x=-3..3,y=-3..3,colour=[BLACK,RED]);`

Click here for how this would look.

3 The Homework Problems (the Only Part to Hand In)

The assignment is to prepare, execute, print, and hand in a Maple worksheet on which the following problems (1)-(6) are done.

(1) Start with the circle equation used in text section 2.4 and make Maple `subs` give you a new equation, `New1` by replacing all occurrences of y by iy. Recall that Maple uses `I` for the imaginary unit i.

(2) Use `implicitplot` to plot both the circle (black) and `New1` (red) on the same plot. Annotate this plot with the name of the type of curve `New1` is.

(3) The `subs` command is more dangerous than you might think. Again, use `subs` to replace all occurrences of **4** in the text circle equation by **1/10**. And then, replace all occurrences of $\boldsymbol{y}$ by $\boldsymbol{iy}$ as above. Call the resulting equation `New2`. And plot `New1` (red) and `New2` (cyan) together on one plot.

(4) This time get equation `New3` by using `subs` to replace all occurrences of **4** in equation `New1` by **0**. Think about factoring `New3` to predict its graph. Write down your prediction on the worksheet. Then issue

```
implicitplot(New3,x=-2..2,y=-10..10);
```

This shows that `implicitplot` is attempting something which is hard to do.

Leaving the above `implicitplot` in your worksheet, issue also an improved version by inserting new options, and perhaps correcting the given ones, in order to get a plot conforming to your predictions.

(5) Use a `subs` and `implicitplot` seqnence to show a graph on which we have both `New1`, and `New4`, which is `New1` shifted to the right by one unit and upward (that is, in the $+\boldsymbol{y}$ direction) by two units.

(6) Finally, get `New5` by issuing

(i) `subs(x=u,y=v, New1);`

(ii) `New5 := subs(u=y,v=x, %);`

Write a prediction as to the relation of the graphs of `New1` and `New5` into your worksheet.

Plot `New1` (red) and `New5` (black) on the same graph with range large enough that you can see both graphs.