Wed Jul 27 20:40:47 MDT 2005 /m170.sp05/handouts170/t4_170_429/key4_170sp05_429

This was a No-Calculators Test

1 Rewrite $g(x)$ in terms of calculus-friendly rational exponents:

$$g(x) = 24x^{1/3} + 25x^{2/3}.$$

Then the most general antiderivative is

$$G(x) = \int g(x)\,dx = 18x^{4/3} + 15x^{5/3} + C.$$

2 We have

$$g(x) = \int (6x - 2\sin(x))\,dx = 3x^2 + 2\cos(x) + C.$$

Using $g(0) = -7$ to evaluate C yields

$$g(x) = 3x^2 + 2\cos(x) - 9.$$

3 (a) Two L'Hôpital's Rule invocations:

$$\begin{aligned} \lim_{x\to 0} \frac{e^x - 1 - x}{x^2} &= \lim_{x\to 0} \frac{e^x - 1}{2x} \\ &= \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2} \end{aligned}$$

(b) To evaluate $\lim\limits_{x\to 0+} \left(1 + 10x + 3x^2\right)^{1/x}$, we resort to the Old Logarithm Trick. We study the affiliated limit:

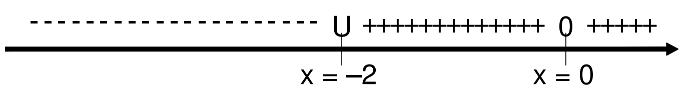
$$\begin{aligned} \lim_{x\to 0+} \ln\left[\left(1 + 10x + 3x^2\right)^{1/x}\right] &= \lim_{x\to 0+} \frac{1}{x} \ln\left[\left(1 + 10x + 3x^2\right)\right] \\ &= \lim_{x\to 0+} \frac{\ln\left(1 + 10x + 3x^2\right)}{x} \\ &= \lim_{x\to 0+} \frac{\left(\dfrac{10 + 6x}{1 + 10x + 3x^2}\right)}{1} = 10 \end{aligned}$$

Thus the original limit has value e^{10}.

4 Fill in the table of antiderivatives:

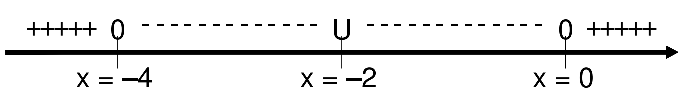
Function	Most General Antiderivative	Function	Most General Antiderivative
x^2	$\boxed{\dfrac{x^3}{3}+C}$	$\dfrac{1}{x^2}$	$\boxed{-x^{-1}+C}$
$\sin(x)$	$\boxed{-\cos(x)+C}$	$\cos(x)$	$\boxed{\sin(x)+C}$
$\sqrt{x}$	$\boxed{\dfrac{2}{3}x^{3/2}+C}$	e^{3x}	$\boxed{\dfrac{e^{3x}}{3}+C}$
$\sec(x)\tan(x)$	$\boxed{\sec(x)+C}$	$\sec(x)^2$	$\boxed{\tan(x)+C}$
$\dfrac{1}{1+x^2}$	$\boxed{\arctan(x)+C}$	$\dfrac{1}{\sqrt{1-x^2}}$	$\boxed{\arcsin(x)+C}$
$\dfrac{1}{x}$	$\boxed{\ln(\lvert x\rvert)+C}$	$\sqrt[5]{x^3}$	$\boxed{\dfrac{5}{8}x^{8/5}+C}$

5 For $f(x)=\dfrac{x^2}{x+2}$ we have



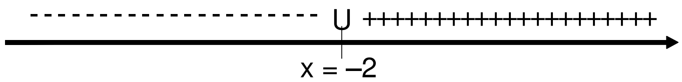
which shows a vertical asymptote at $x=-2$ and an x-intercept at $x=0$, which must be a local minimum.

And for $f'(x)=\dfrac{x\,(x+4)}{(x+2)^2}$ we have



which seconds the previous assertion that f has a local minimum at $x=0$ and also shows a local maximum at $x=-4$.

And, finally, for $\boldsymbol{f''(x) = \dfrac{8}{(x+2)^3}}$ we have



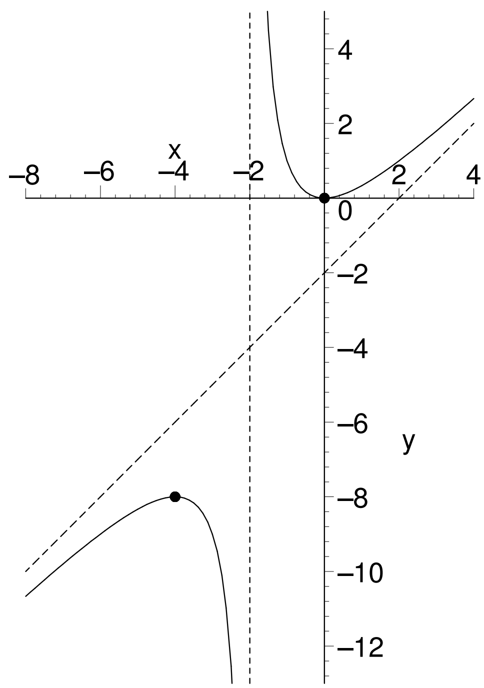
which shows that, although concavity differs from place to place on $\boldsymbol{f}$, there are no inflection points.

Long division reveals a further formula for $\boldsymbol{f}$:

$$\boldsymbol{f(x) = x - 2 + \frac{4}{x+2}}.$$

This shows that $\boldsymbol{y = x - 2}$ is an oblique asymptote for the curve.

Here is a machine-generated graph showing the asymptotes, the maximum at $\boldsymbol{(-4,-8)}$, the minimum at $\boldsymbol{(0,0)}$, and the concavity:

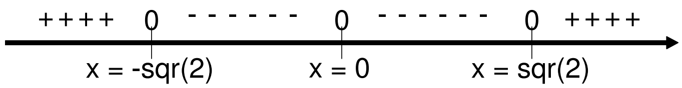


6 Sketch a assignment-#37-style graph of the function $\boldsymbol{f(x) = 3x^5 - 10x^3}$.

For $\boldsymbol{f(x) = 3x^5 - 10x^3}$, we have

$$\boldsymbol{f'(x) = 15x^2(x+\sqrt{2})(x-\sqrt{2})}$$

for which the sign chart is

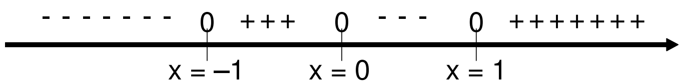


showing rising from $-\infty$ to a maximum at $\boldsymbol{x = -\sqrt{2}}$, then down to a shoulder at $\boldsymbol{x = 0}$, then down some more to a minimum at $\boldsymbol{x = \sqrt{2}}$, and finally rises to $+\infty$.

Further, we have

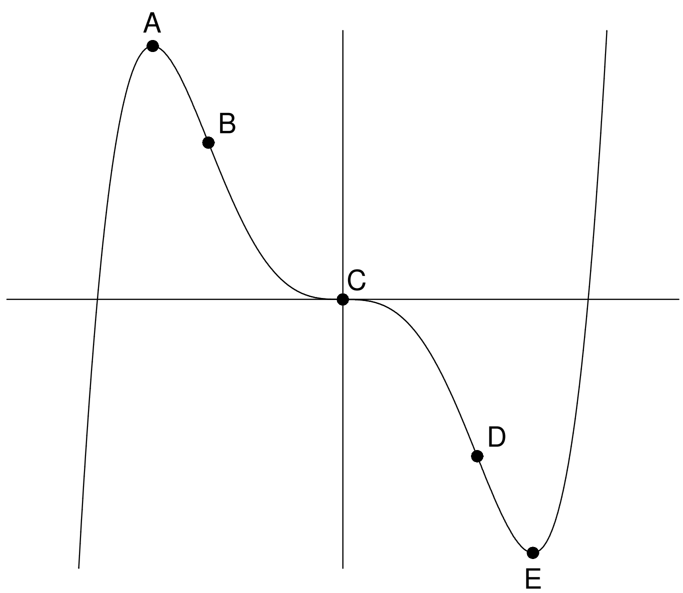
$$\boldsymbol{f''(x) = 60x(x+1)(x-1)}$$

for which the sign chart is



showing downward concavity from $-\infty$ up to $\boldsymbol{x = -1}$, upward concavity up to $\boldsymbol{x = 0}$, then downward concavity until $\boldsymbol{x = 1}$, then upward concavity thereafter.

Here is a machine-generated graph showing the extremes and inflections labeled with letters linking to a table of coordinates



A	$\boldsymbol{(-\sqrt{2}, 8\sqrt{2})}$
B	$\boldsymbol{(-1, 7)}$
C	$\boldsymbol{(0, 0)}$
D	$\boldsymbol{(1, -7)}$
E	$\boldsymbol{(\sqrt{2}, -8\sqrt{2})}$